

Topic : Newton-Raphson Method - Roots of Equations

Simulation : Convergence of the Method

Language : Mathematica 4.1

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Abstract : This simulation illustrates the convergence of the Newton-Raphson method of finding the root of an equation  $f[x] = 0$ .

### ■ INPUTS: Enter the Following

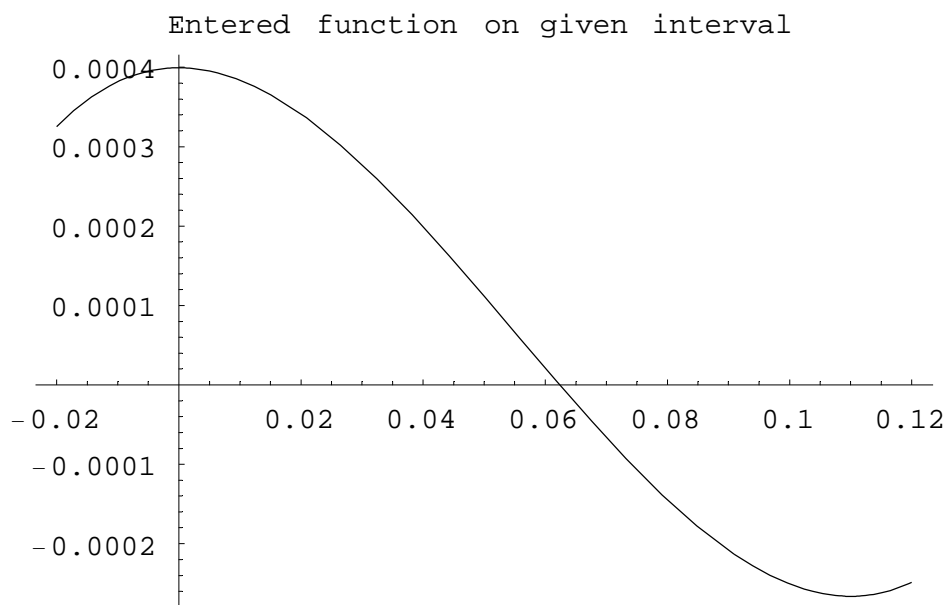
Function in  $f[x] = 0$

```
In[44]:= f[x_] := x^3 - 0.165 * x^2 + 3.993 * 10^-4
```

Range of 'x' you want to see the function

```
In[45]:= xb = -0.02;  
xe = 0.12;
```

```
In[47]:= curve = Plot[f[x], {x, xb, xe}, PlotLabel ->  
"Entered function on given interval", TextStyle -> {FontSize -> 11}];
```



Initial guess

```
In[48]:= x0 = 0.05;
```

Maximum number of iterations

```
In[49]:= nmaximum = 4;
```

Counting from the left, enter the number of the root desired

```
In[50]:= numroot = 2;
```

Because this method uses a line tangent to the function at the initial guess, we must calculate the derivative of the function to find the slope of the line at this point. Here we will define the derivative of the function  $f(x)$  as  $g(x)$ .

```
In[51]:= g[x_] := f'[x]
```

```
In[52]:= $MinPrecision = 20;
```

## ■ True Solution

This is the solution found by Mathematica

```
In[53]:= xactual = Root[f[x], numroot]
```

```
Out[53]= 0.0623776
```

## ■ Value of root as a function of iterations

Here the Newton-Raphson method algorithm is applied to generate the values of the roots, threue error, absolute relative true error, approximate error, absolute relative approximate error, and the number of significant digits at least correct in the estimated root as a function of number of iterations.

```
In[54]:= Clear[xr];
```

```
In[55]:= SetPrecision[xr, 40]
```

```
Out[55]= xr
```

```
In[56]:= Array[xr, nmaximum];
```

```
In[57]:= For[i = 1; xr[0] = x0, i <= nmaximum, i++,
  xr[i] = xr[i - 1] - f[xr[i - 1]] / g[xr[i - 1]]]
```

## ■ Absolute true error

```
In[58]:= Array[Et, nmaximum];
```

```
In[59]:= For[i = 1, i <= nmaximum, i++, Et[i] = Abs[xactual - xr[i]]]
```

## ■ Absolute relative true error

```
In[60]:= Array[et, nmaximum];
```

```
In[61]:= For[i = 1, i <= n_maximum, i++, et[i] = Abs[Et[i] / xactual * 100]]
```

## ■ Absolute approximate error

```
In[62]:= Array[Ea, n_maximum];
```

```
In[63]:= For[i = 1, i <= n_maximum, i++,  
  If[i <= 1, Ea[i] = Abs[xr[i] - x0], Ea[i] = Abs[xr[i] - xr[i - 1]]]]
```

## ■ Absolute relative approximate error

```
In[64]:= Array[ea, n_maximum];
```

```
In[65]:= For[i = 1, i <= n_maximum, i++, ea[i] = Abs[Ea[i] / xr[i] * 100]]
```

## ■ Significant digits at least correct

```
In[66]:= Array[sigdig, n_maximum];
```

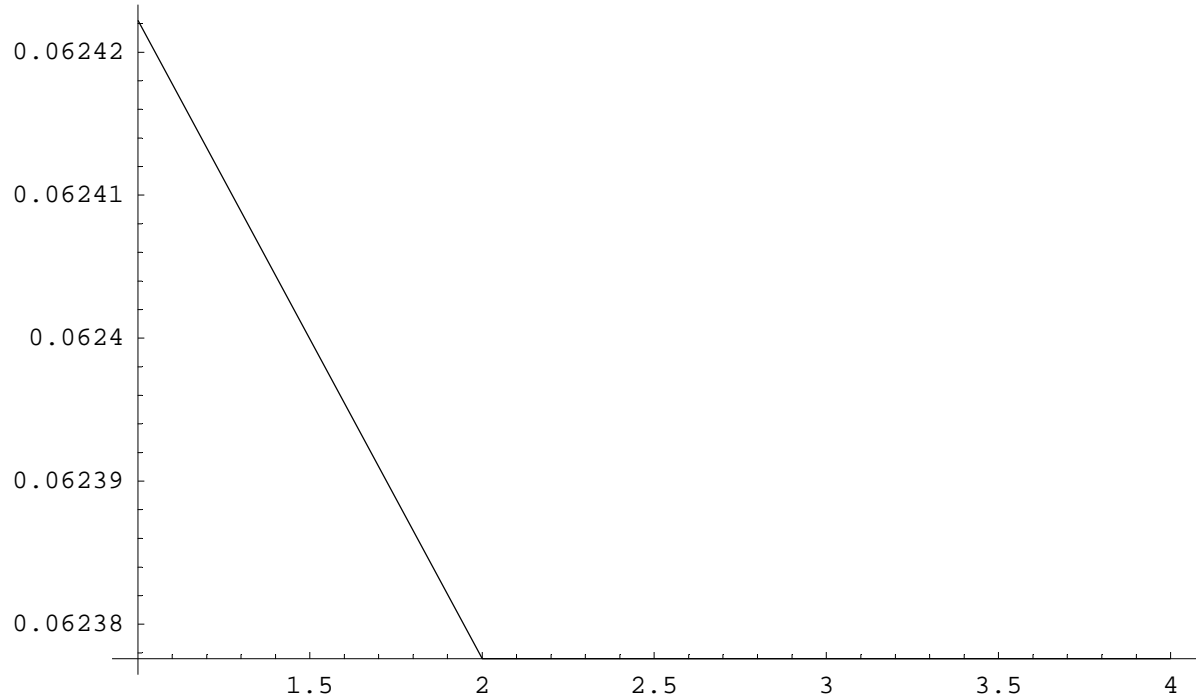
```
In[67]:= For[i = 1, i <= n_maximum, i++, If[(ea[i] ≥ 5) || (i <= 1),  
  sigdig[i] = 0, sigdig[i] = Floor[(2 - Log[10, Abs[ea[i] / 0.5]])]]]
```

## ■ Graphs

```
In[68]:= xrplot = Table[xr[i], {i, 1, n_maximum}];
```

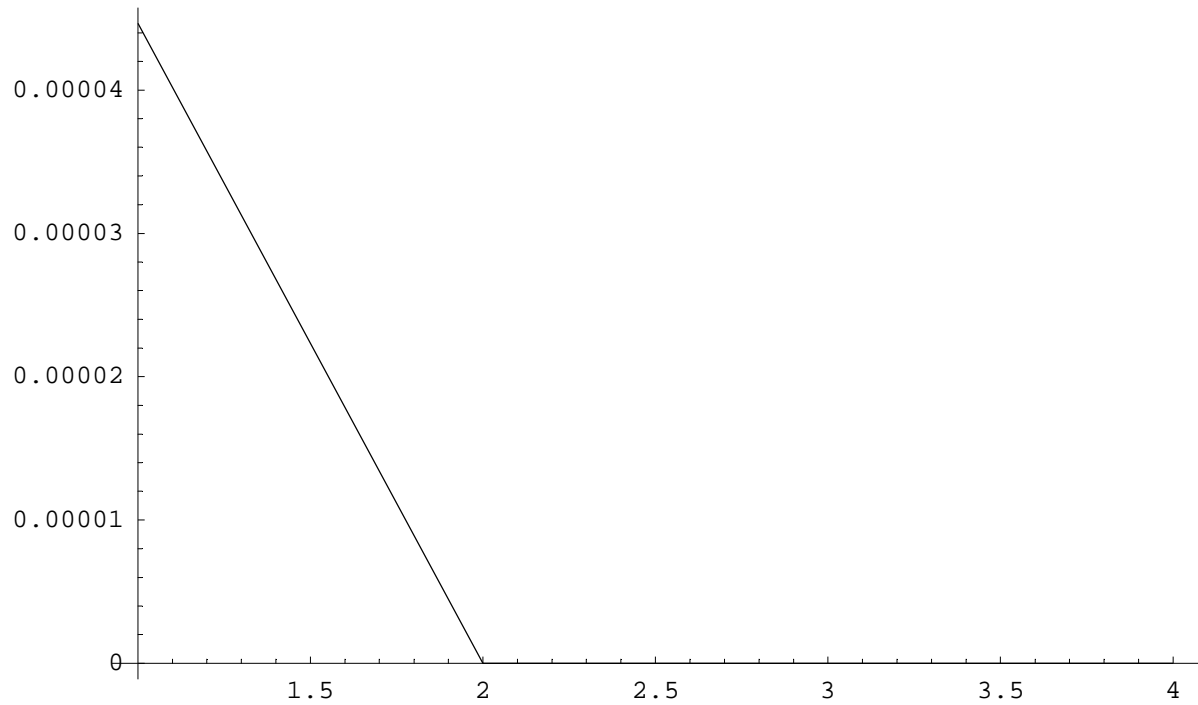
```
In[69]:= ListPlot[xrplot, PlotJoined -> True,  
  PlotRange -> All, AxesOrigin -> {1, Min[xrplot]},  
  PlotLabel -> "Estimated root as a function of number of iterations"];
```

Estimated root as a function of number of iterations



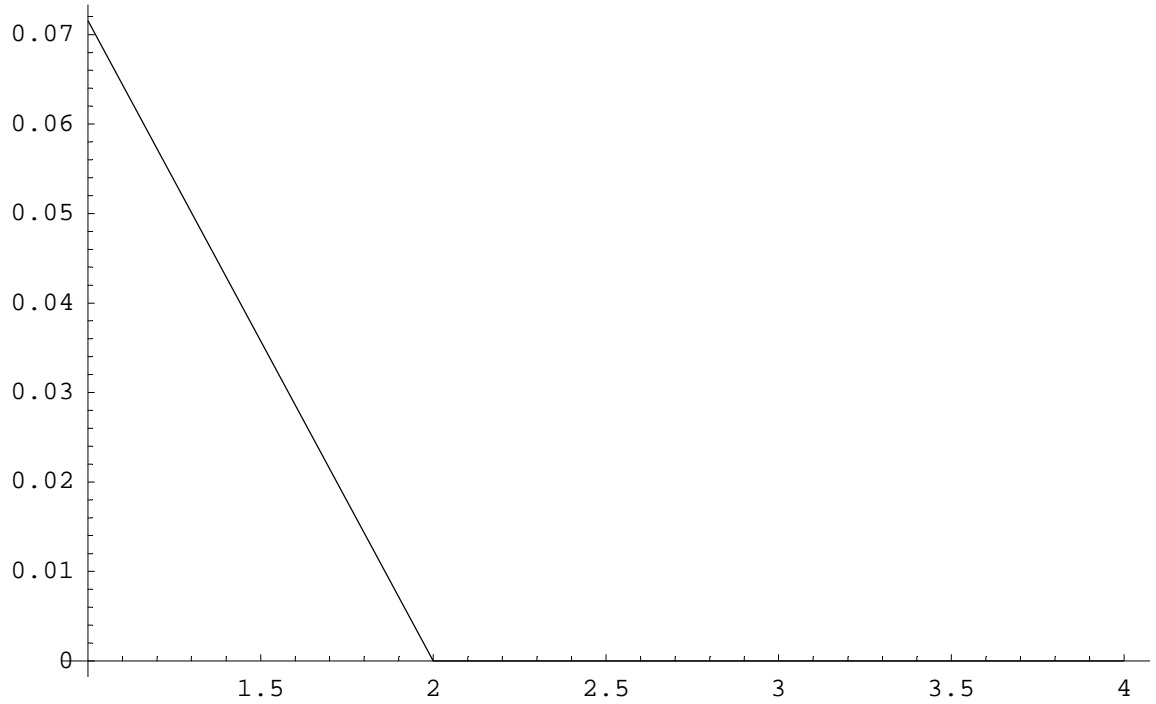
```
In[70]:= Etplot = Table[Et[i], {i, 1, nmaximum}];
```

```
In[71]:= ListPlot[Etplot, PlotJoined -> True,  
  PlotRange -> All, AxesOrigin -> {1, Min[Etplot]},  
  PlotLabel -> "Absolute true error as a function of number of iterations";  
  Absolute true error as a function of number of iterations
```



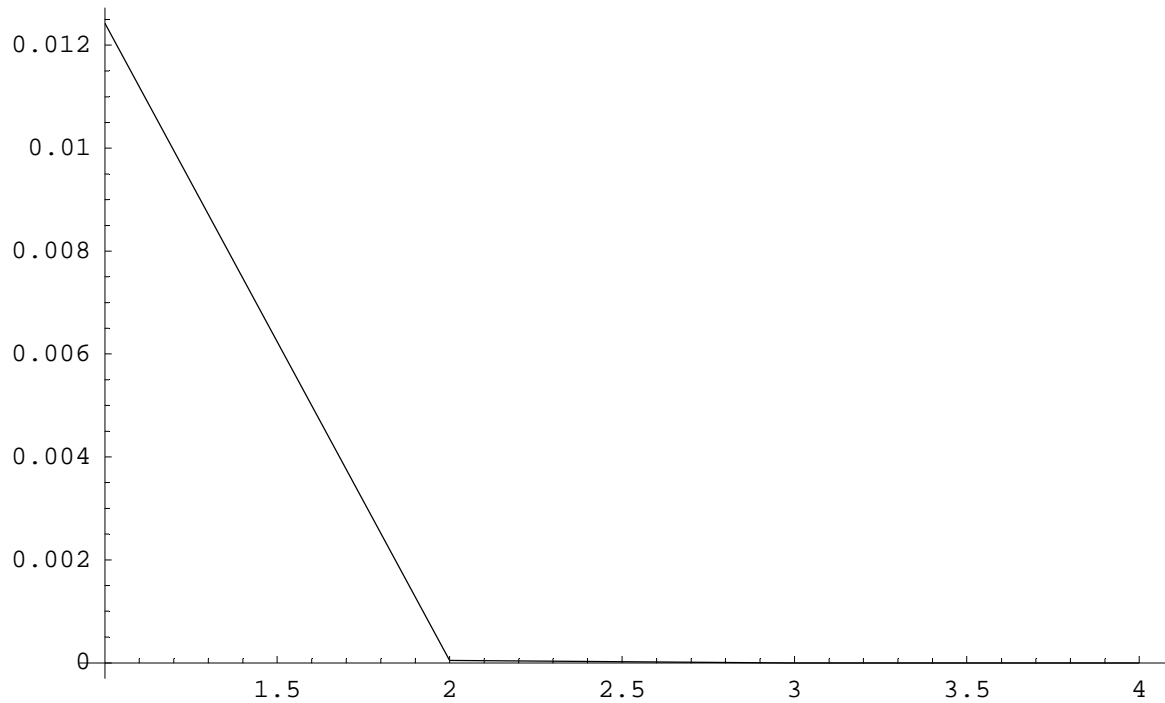
```
In[72]:= etplot = Table[et[i], {i, 1, nmaximum}]
```

```
In[73]:= ListPlot[etplot, PlotJoined -> True,  
PlotRange -> All, AxesOrigin -> {1, Min[etplot]}, PlotLabel ->  
"Absolute relative true error as a function of number of iterations";  
Absolute relative true error as a function of number of iterations
```



```
In[74]:= Eaplot = Table[Ea[i], {i, 1, nmaximum}];
```

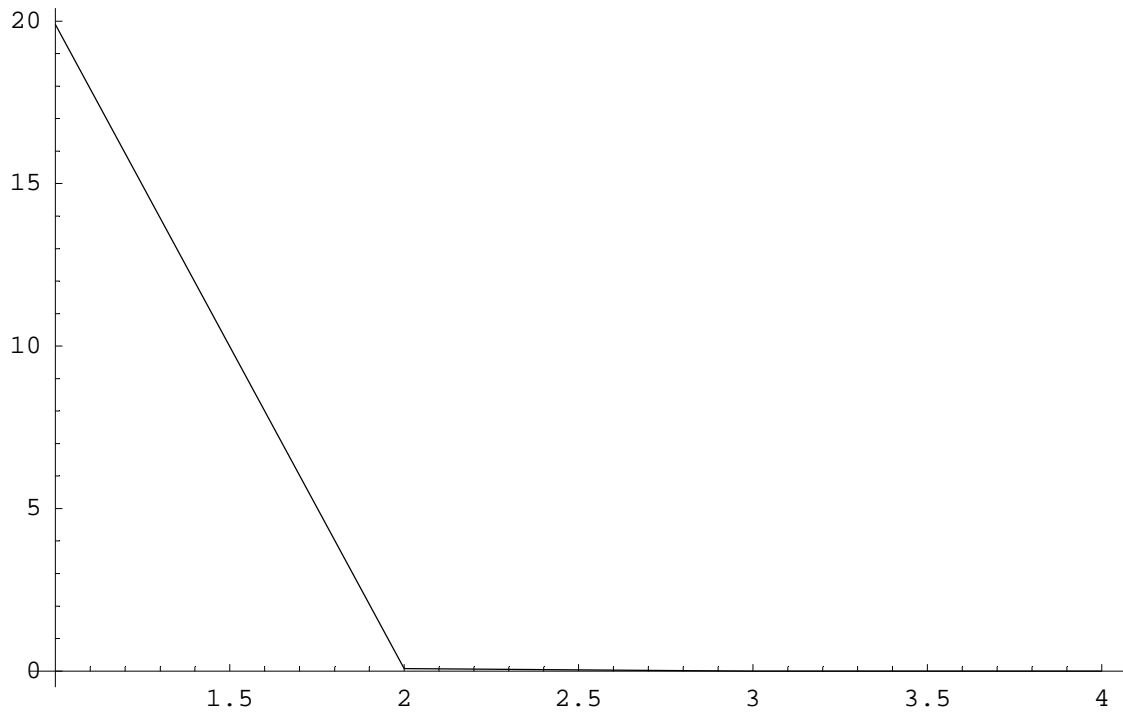
```
In[75]:= ListPlot[Eaplot, PlotJoined -> True,  
PlotRange -> All, AxesOrigin -> {1, Min[Eaplot]}, PlotLabel ->  
"Absolute approximate error as a function of number of iterations";  
Absolute approximate error as a function of number of iterations
```



```
In[76]:= eaplot = Table[ea[i], {i, 1, nmaximum}]
```

```
In[77]:= ListPlot[εaplot, PlotJoined → True,  
PlotRange → All, AxesOrigin → {1, Min[εaplot]},  
PlotLabel → "Absolute relative approximate error  
as a function of number of iterations"];
```

Absolute relative approximate error as a function of number of iterations



```
In[81]:= sigdigplot = Table[sigdig[i], {i, 1, nmaximum}];
```

```
In[79]:= << Graphics`Graphics`
```



```
In[80]:= BarChart[sigdigplot];
```

