Topic : Secant Method - Roots of Equations
Simulation : Convergence of the Method
Language : Mathmatica 4.1
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Abstract : This simulation illustrates the convergence of the secant method of finding the root of an equation $f[x]$ 0.

■ INPUTS: Enter the Following

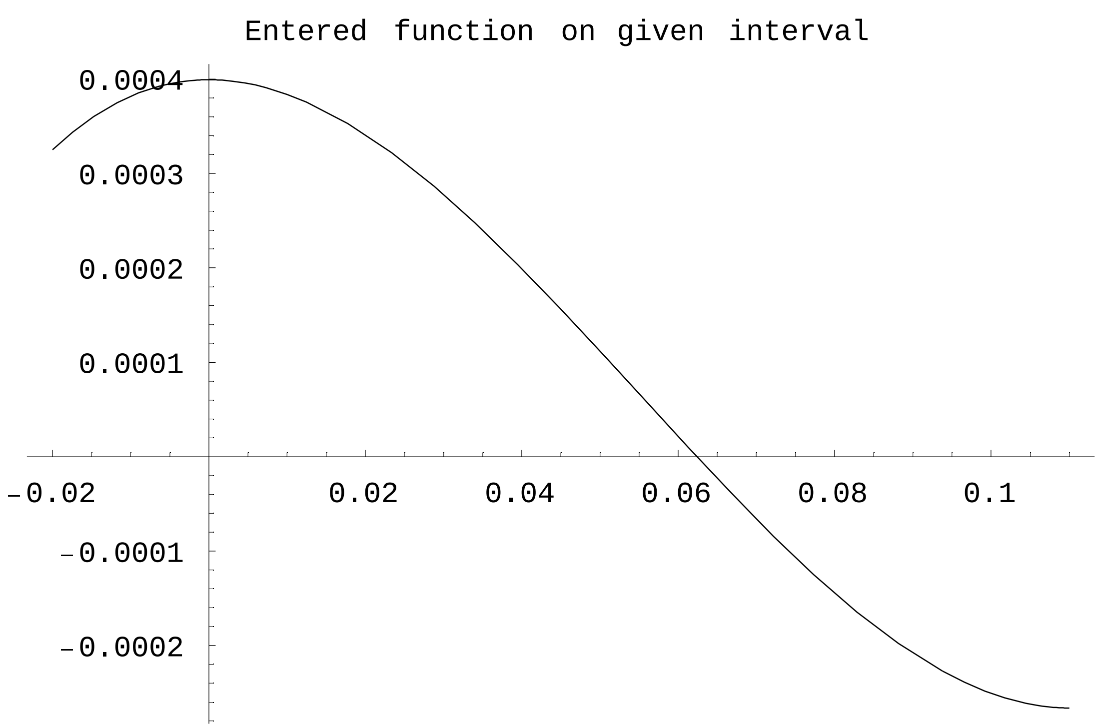
Function in $f[x]$ 0

```
In[90]:= f[x_] := x^3 - 0.165 * x^2 + 3.993 * 10^-4
```

Range of 'x' you want to see the function

```
In[91]:= x_b = -0.02;
         x_e = 0.11;
```

```
In[93]:= curve = Plot[f[x], {x, x_b, x_e}, PlotLabel →
            "Entered function on given interval", TextStyle → {FontSize → 11}];
```



First initial guess

In[94]:=
```
xi1 = 0.02;
```

Second initial guess

In[95]:=
```
xi2 = 0.05;
```

Maximum number of iterations

In[96]:=
```
nmaximum = 5;
```

Counting from the left, enter the number of the root desired

In[97]:=
```
numroot = 2;
```

■ SOLUTION

In[98]:=
```
maxi = f[xb];
mini = f[xb];
step = (xe - xb) / 10;
Do[ If[f[i] > maxi, maxi = f[i]];
  If[f[i] < mini, mini = f[i]], {i, xb, xe, step}];
tot = maxi - mini;
mini = mini - 0.1 * tot;
maxi = maxi + 0.1 * tot;
```

Check first if the lower and upper guesses bracket the root of the equation

In[105]:=
```
f[xi1]
```

Out[105]= 0.0003413

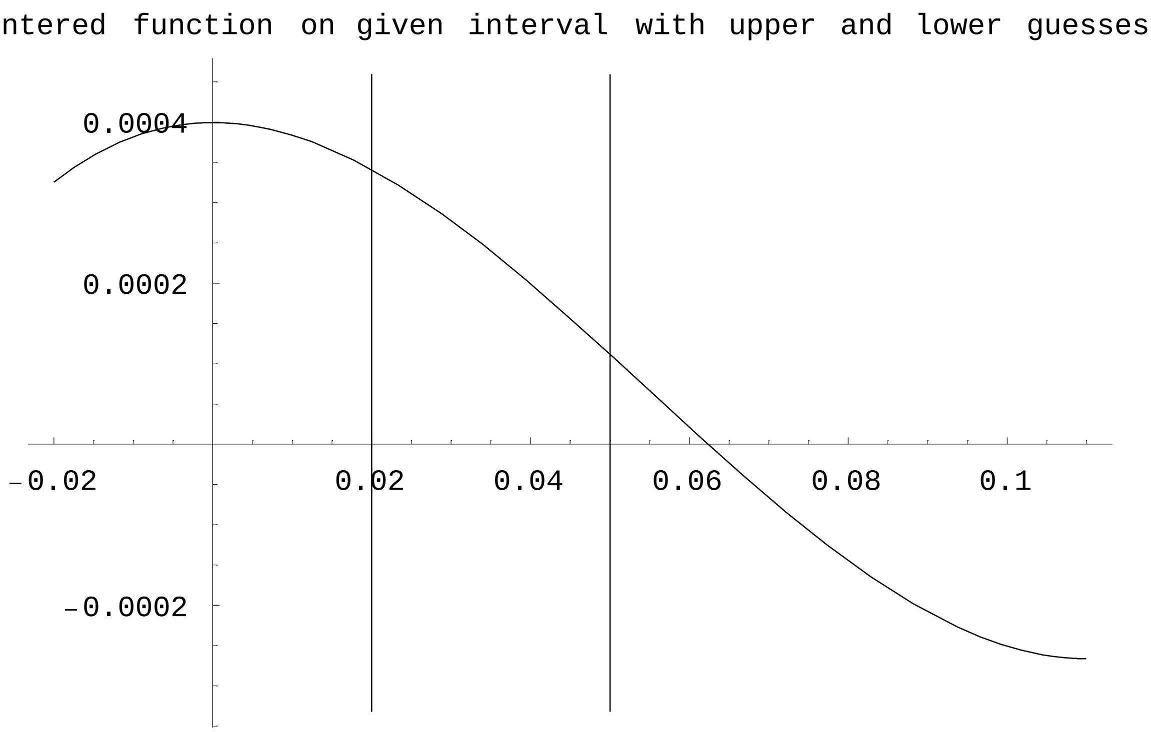
In[106]:=
```
f[xi2]
```

Out[106]= 0.0001118

In[107]:=
```
% * %%
```

Out[107]= 3.81573×10^{-8}

In[108]:=
```
Show[Graphics[Line[{{xi1, maxi}, {xi1, mini}}]], curve,
  Graphics[Line[{{xi2, maxi}, {xi2, mini}}]], Axes → True, PlotLabel →
   "Entered function on given interval with upper and lower guesses",
  TextStyle → {FontSize → 11}];
```



■ True Solution

This is the solution found by Mathematica

In[109]:=
```
xactual = Root[f[x], numroot]
```

Out[109]= 0.0623776

■ Value of root as a function of iterations

Here the bisection method algorithm is applied to generate the values of the roots, thrue error, absolute relative true error, approximate error, absolute relative approximate error, and the number of significant digits at least correct in the estimated root as a function of number of iterations.

In[110]:=
```
Array[xr, n_maximum];
```

In[111]:=
```
For[i = 1; x1 = xi1; x2 = xi2, i <= n_maximum, i++,
  xr[i] = x2 - (f[x2] * (x1 - x2)) / (f[x1] - f[x2]); x1 = x2; x2 = xr[i]]
```

■ Absolute true error

In[112]:=
```
Array[Et, n_maximum];
```

In[113]:=
```
For[i = 1, i <= n_maximum, i++, Et[i] = Abs[xactual - xr[i]]]
```

■ Absolute relative true error

In[114]:=
```
Array[ϵt, n_maximum];
```

In[115]:=
```
For[i = 1, i <= n_maximum, i++, ϵt[i] = Abs[Et[i] / xactual * 100]]
```

■ Absolute approximate error

In[116]:=
```
Array[Ea, n_maximum];
```

In[117]:=
```
For[i = 1, i <= n_maximum, i++,
  If[i <= 1, Ea[i] = Abs[xr[i] - xi2], Ea[i] = Abs[xr[i] - xr[i - 1]]]]
```

■ Absolute relative approximate error

In[118]:=
```
Array[ϵa, n_maximum];
```

In[119]:=
```
For[i = 1, i <= n_maximum, i++, ϵa[i] = Abs[Ea[i] / xr[i] * 100]]
```

■ Significant digits at least correct

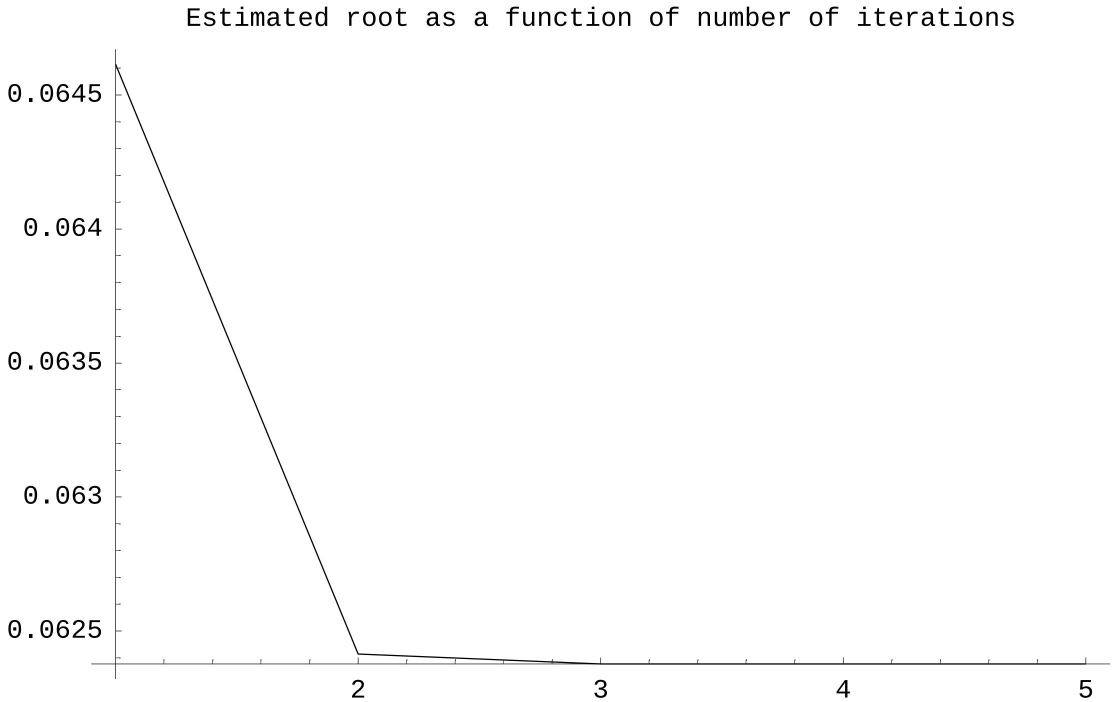
In[120]:=
```
Array[sigdig, n_maximum];
```

In[121]:=
```
For[i = 1, i <= n_maximum, i++, If[(ϵa[i] ≥ 5) || (i <= 1),
   sigdig[i] = 0, sigdig[i] = Floor[(2 - Log[10, Abs[ϵa[i] / 0.5]])]]]
```

■ Graphs

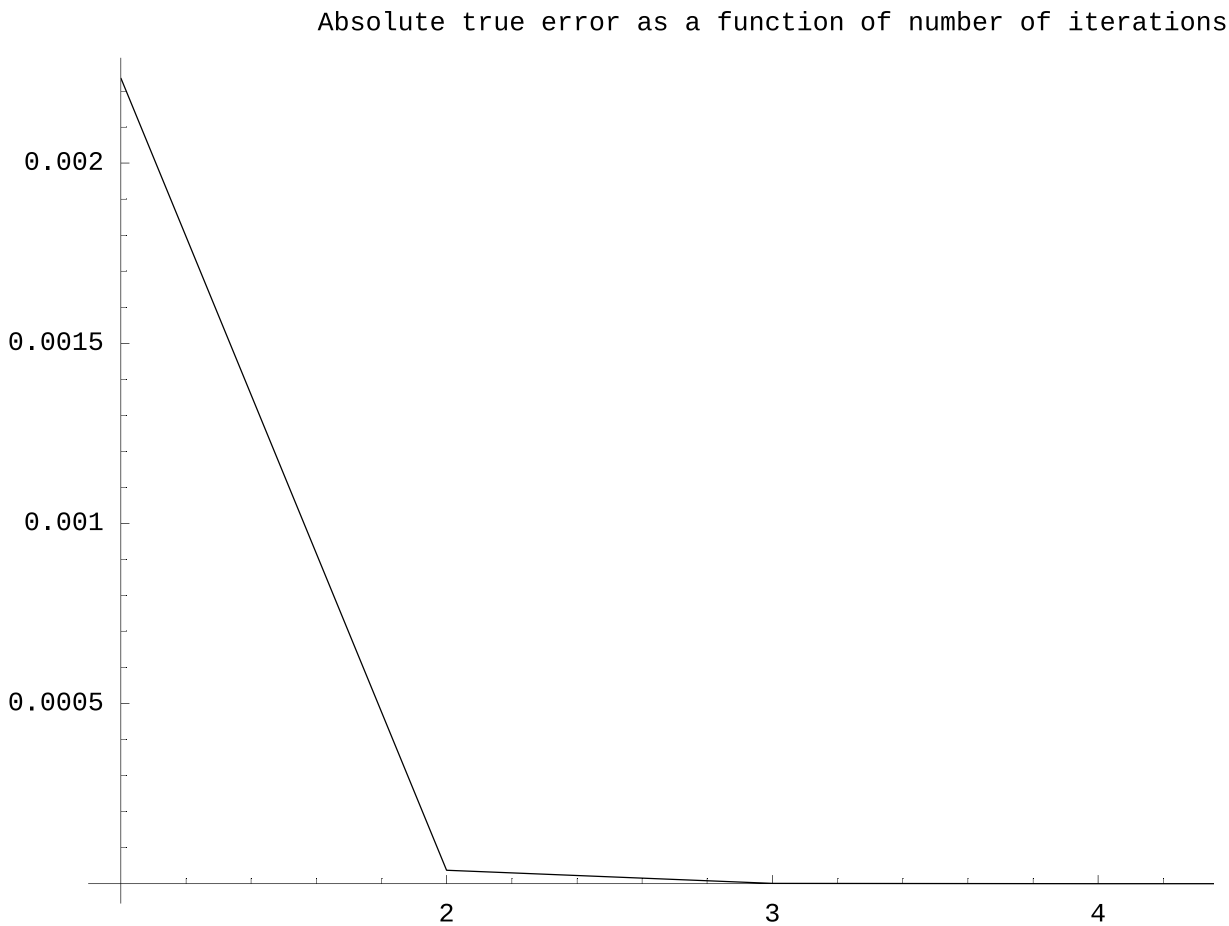
In[122]:=
```
xrplot = Table[xr[i], {i, 1, n_maximum}];
```

In[123]:=
```
ListPlot[xrplot, PlotJoined → True,
 PlotRange -> All, AxesOrigin -> {1, Min[xrplot]},
 PlotLabel -> "Estimated root as a function of number of iterations"];
```



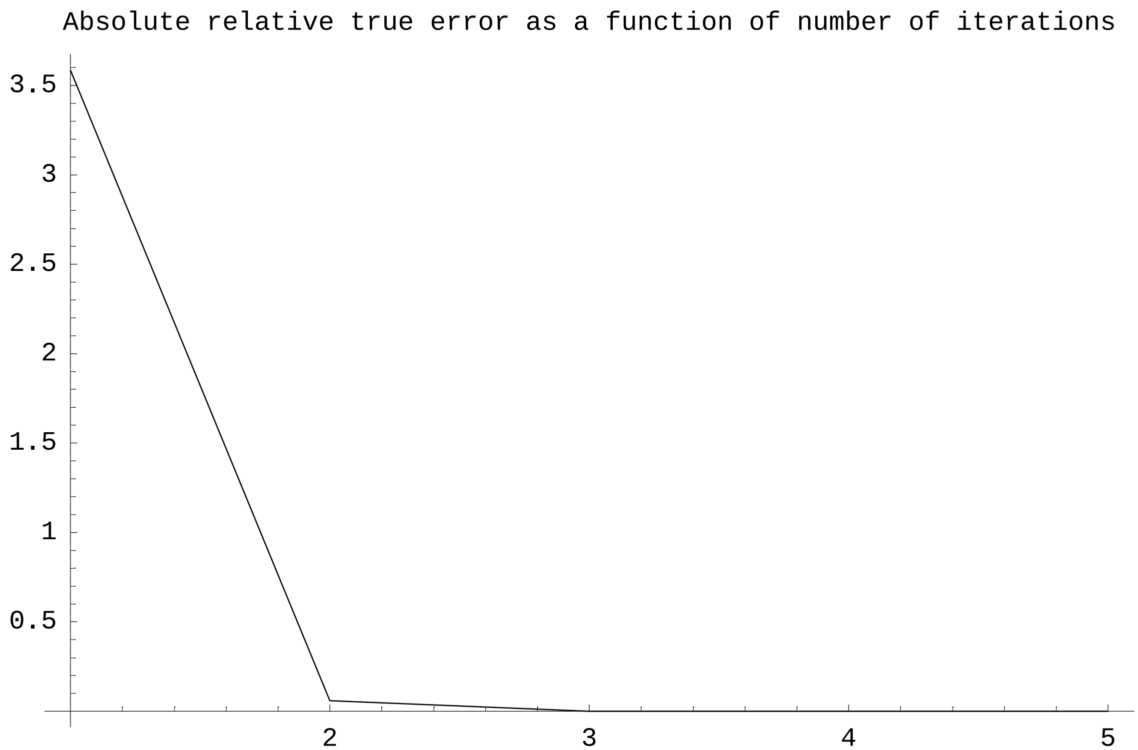
In[124]:=
```
Etplot = Table[Et[i], {i, 1, n_maximum}];
```

In[125]:=
```
ListPlot[Etplot, PlotJoined → True,
 PlotRange -> All, AxesOrigin -> {1, Min[Etplot]},
 PlotLabel -> "Absolute true error as a function of number of iterations"];
```



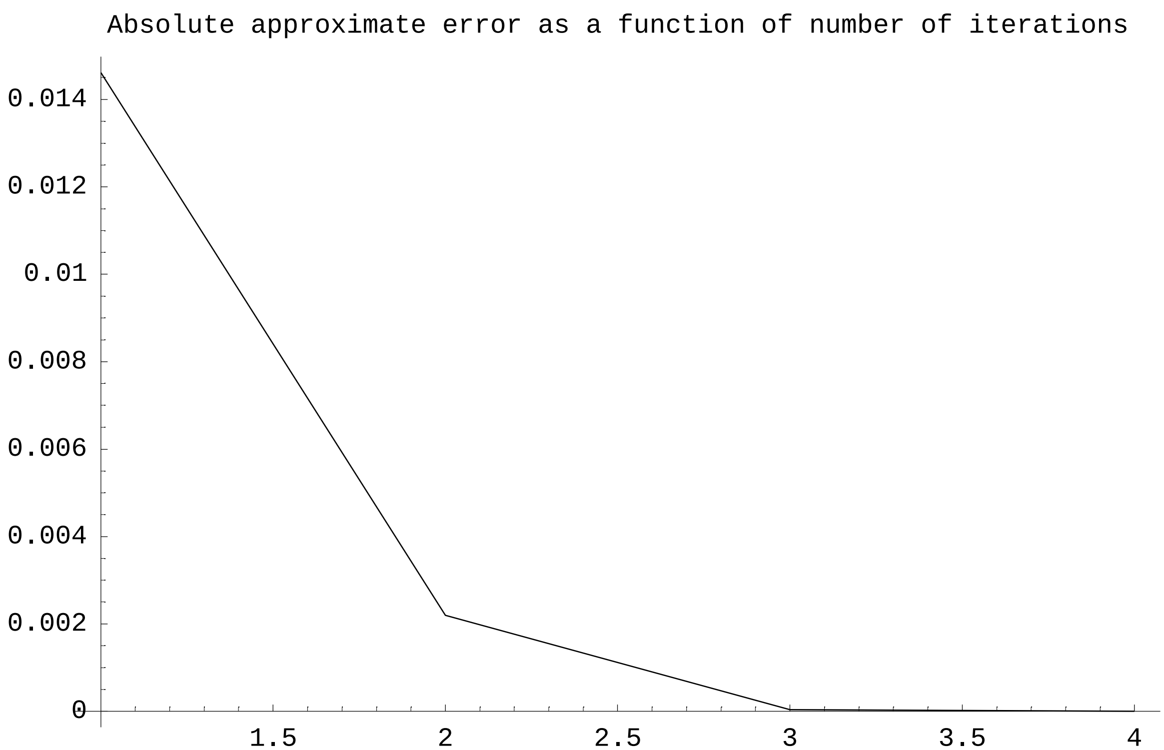
In[126]:=
```
ϵtplot = Table[ϵt[i], {i, 1, n_maximum}];
```

In[127]:=
```
ListPlot[ϵtplot, PlotJoined → True,
 PlotRange -> All, AxesOrigin -> {1, Min[ϵtplot]}, PlotLabel ->
  "Absolute relative true error as a function of number of iterations"];
```



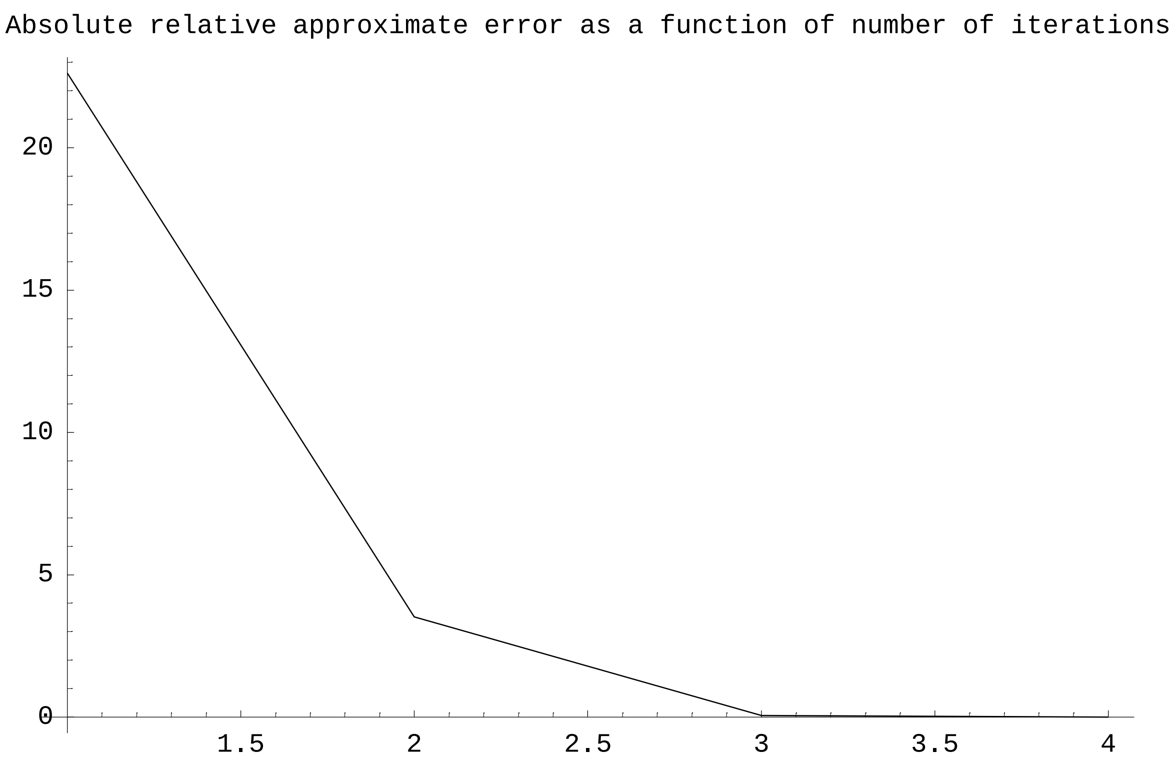
In[128]:=
```
Eaplot = Table[Ea[i], {i, 1, n_maximum - 1}];
```

In[129]:=
```
ListPlot[Eaplot, PlotJoined → True,
  PlotRange -> All, AxesOrigin -> {1, Min[Eaplot]}, PlotLabel ->
   "Absolute approximate error as a function of number of iterations"];
```



In[130]:=
```
εaplot = Table[εa[i], {i, 1, n_maximum - 1}];
```

```
In[131]:= ListPlot[∈aplot, PlotJoined → True,
  PlotRange -> All, AxesOrigin -> {1, Min[∈aplot]},
  PlotLabel -> "Absolute relative approximate error
    as a function of number of iterations"];
```




```
In[132]:= sigdigplot = Table[sigdig[i], {i, 1, nmaximum}];
```

```
In[133]:= << Graphics`Graphics`
```

In[134]:=
```
BarChart[sigdigplot];
```