

Topic : Secant Method - Roots of Equations

Simulation : Pitfall - Root jumping several roots away

Language : Mathematica 4.1

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Abstract : The following example illustrates how, in the Secant method, an initial guess close to one root can jump to a location several roots away.

■ **INPUTS: Enter the Following**

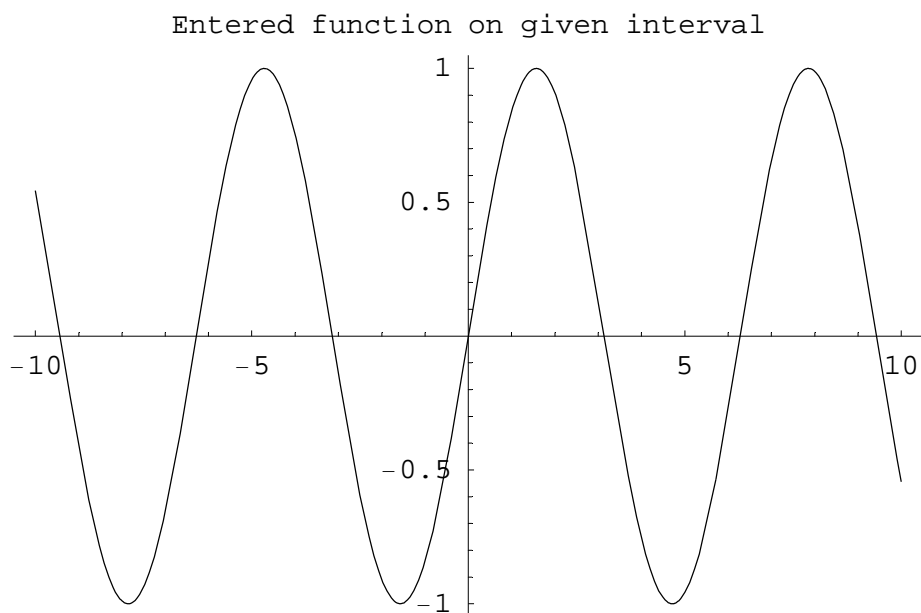
Function in $f[x]$ 0

```
In[175]:= f[x_] := Sin[x]
```

Range of 'x' you want to see the function

```
In[176]:= x_b = -10.0;  
x_e = 10.0;
```

```
In[178]:= curve = Plot[f[x], {x, x_b, x_e}, PlotLabel ->  
"Entered function on given interval", TextStyle -> {FontSize -> 11}];
```



Lower initial guess

```
In[179]:= x_guess1 = 8.5;
```

Upper initial guess

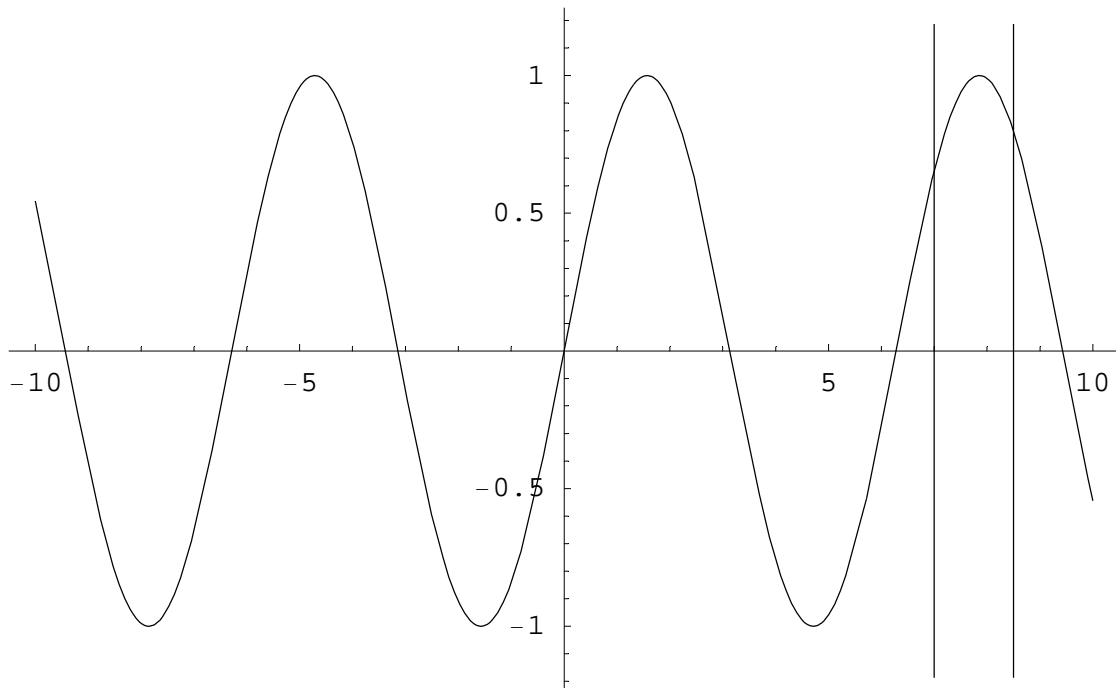
```
In[180]:= xguess2 = 7;
```

■ SOLUTION

```
In[181]:= maxi = f[x_b];
mini = f[x_b];
step = (x_e - x_b) / 10;
Do[ If[f[i] > maxi, maxi = f[i]];
  If[f[i] < mini, mini = f[i]], {i, x_b, x_e, step}];
tot = maxi - mini;
mini = mini - 0.1 * tot;
maxi = maxi + 0.1 * tot;
```

Check first if the lower and upper guesses bracket the root of the equation

```
In[188]:= Show[Graphics[Line[{{xguess2, maxi}, {xguess2, mini}}]], curve,
Graphics[Line[{{xguess1, maxi}, {xguess1, mini}}]], Axes → True, PlotLabel →
"Entered function on given interval with upper and lower guesses",
TextStyle → {FontSize → 11}];
Entered function on given interval with upper and lower guesses
```



Iteration 1

Choose two initial guesses of root.

```
In[189]:= x1' = xguess1;  
          x0 = xguess2;
```

Estimate of root

```
In[191]:= x1 = x0 - (f[x0] * (x1' - x0)) / (f[x1'] - f[x0])  
Out[191]= 0.035503
```

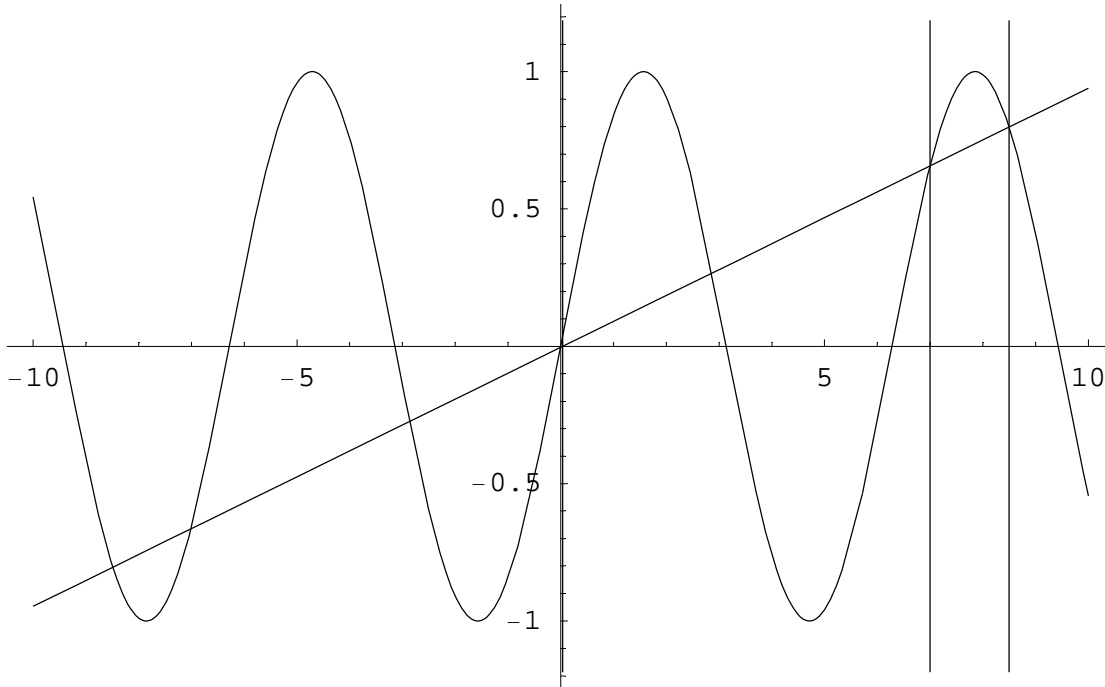
Absolute relative approximate error

```
In[192]:= εa = Abs[(x1 - x0) / x1 * 100]  
Out[192]= 19616.6
```

```
In[193]:= m = (f[x0] - f[x1']) / (x0 - x1');  
          secantline[x_] = m * x + f[x0] - m * x0;
```

```
In[195]:= sline = Plot[secantline[x], {x, xb, xe}];
```

```
In[196]:= Show[Graphics[Line[{{x0, maxi}, {x0, mini}}]],
Graphics[Line[{{x1, maxi}, {x1, mini}}]], curve,
Graphics[Line[{{x1', maxi}, {x1', mini}}]], sline, Axes → True,
PlotLabel → "Entered function on given interval with upper and
lower guesses and estimated root", TextStyle → {FontSize → 11}];
l function on given interval with upper and lower guesses and estimated
```



It should be noted that these two guesses do not have to bracket the root. We have called the two initial guesses in the way which we have because that will be the format for subsequent iterations. It does not matter which guess is which. Try switching the two initial guesses to see what happens. One should converge faster than the other.

Iteration 2

Estimate of root

```
In[197]:= x2 = x1 - (f[x1] * (x0 - x1)) / (f[x0] - f[x1])
```

```
Out[197]= -0.362264
```

Absolute relative approximate error

```
In[198]:= εa = Abs[(x2 - x1) / x2 * 100]
```

```
Out[198]= 109.8
```

```

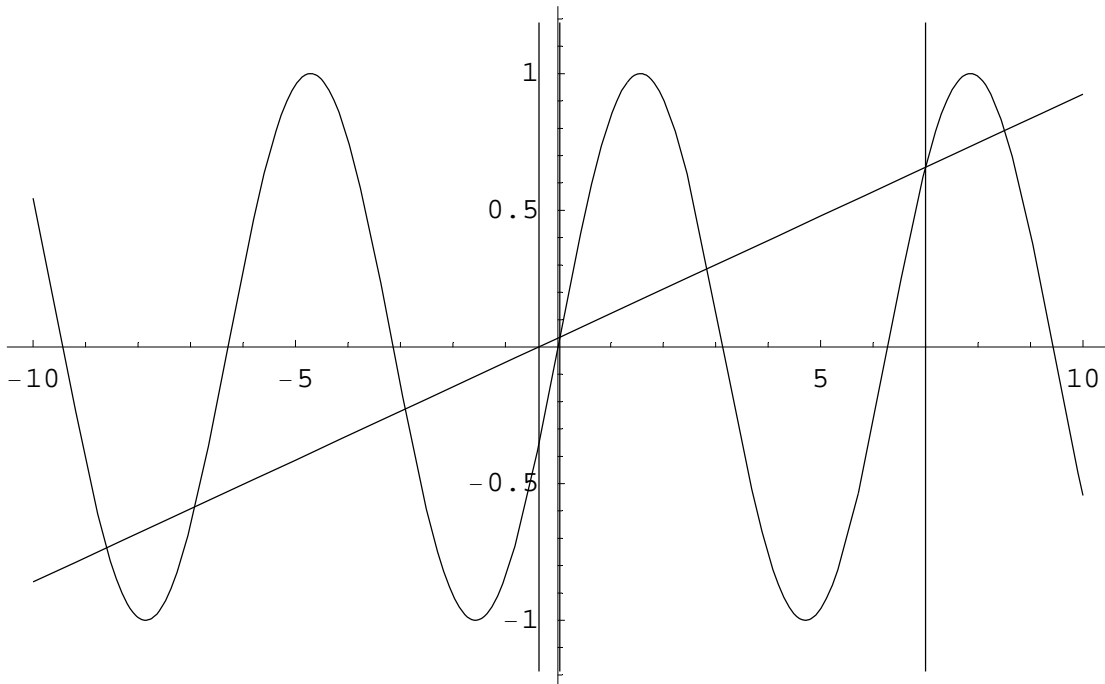
In[199]:= m = (f[x1] - f[x0]) / (x1 - x0);
          secantline[x_] = m * x + f[x1] - m * x1;

In[201]:= sline = Plot[secantline[x], {x, xb, xe}];

In[202]:= Show[Graphics[Line[{{x1, maxi}, {x1, mini}}]],
              Graphics[Line[{{x2, maxi}, {x2, mini}}]], curve,
              Graphics[Line[{{x0, maxi}, {x0, mini}}]], sline, Axes → True,
              PlotLabel → "Entered function on given interval with upper and
                lower guesses and estimated root", TextStyle → {FontSize → 11}];

```

function on given interval with upper and lower guesses and estimated



Iteration 3

Estimate of root

```
In[203]:= x3 = x2 - (f[x2] * (x1 - x2)) / (f[x1] - f[x2])
```

```
Out[203]= -0.000709876
```

Absolute relative approximate error

```
In[204]:=  $\epsilon_a = \text{Abs}[(x_3 - x_2) / x_3 * 100]$ 
```

```
Out[204]= 50932.1
```

```
In[205]:=  $m = (f[x_2] - f[x_1]) / (x_2 - x_1);$   

secantline[x_] = m * x + f[x_2] - m * x_2;
```

```
In[207]:= sline = Plot[secantline[x], {x, x_b, x_e}];
```

```
In[208]:= Show[Graphics[Line[{{x2, maxi}, {x2, mini}}]],  

Graphics[Line[{{x3, maxi}, {x3, mini}}]], curve,  

Graphics[Line[{{x1, maxi}, {x1, mini}}]], sline, Axes → True,  

PlotLabel → "Entered function on given interval with upper and  

lower guesses and estimated root", TextStyle → {FontSize → 11}];
```

. function on given interval with upper and lower guesses and estimated

