Topic : Additional Interpolation Topics
Simulation : Higher Order Interpolation is a Bad Idea
Language : Mathematica 4.1
Authors : Nathan Collier,Autar Kaw
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Abstract : In 1901, Carl Runge published his work on dangers of higher order interpolation. He took a simple looking function f(x)=1/(1+25x2) on the interval [-1,1]. He took points equidistantly spaced in [-1,1] and interpolated the points with polynomials. He found that as he took more points, the polynomials and the original curve differed considerably.

In[505]:=
```
ClearAll;
```

■ INPUTS: Enter the following

You can make 3 seperate choices for the number of equidistant points in [-1,1]. So if you choose n1=5, you are using a 4th order polynomial to approximate 1/(1+25*x^2).

In[506]:=
```
n1 := 5
```

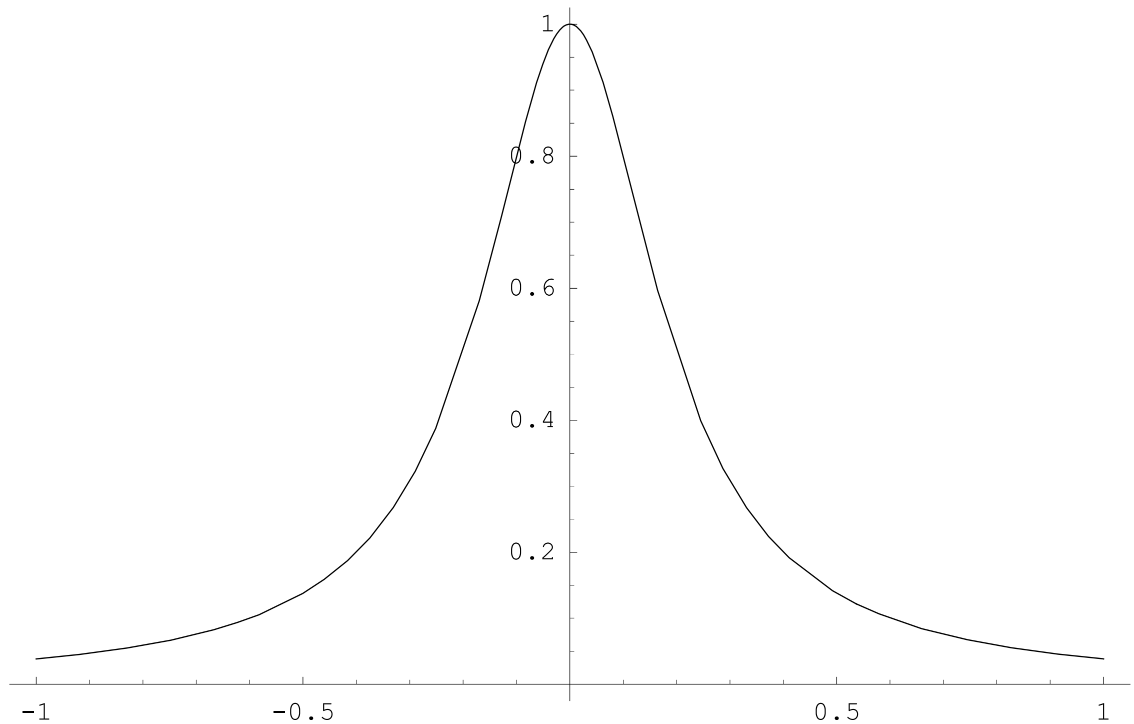
In[507]:=
```
n2 := 9
```

In[508]:=
```
n3 := 17
```

■ SOLUTION

In[509]:=
```
f[x_] := 1 / (1 + 25 * x^2)
```

In[510]:=
```
Runge = Plot[f[x], {x, -1, 1}];
```

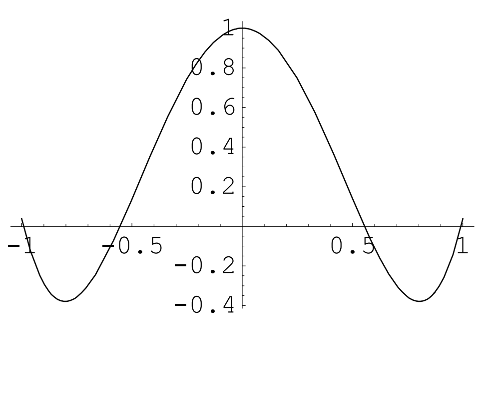


In[511]:=
```
x1 := Table[2 / (n1 - 1) * i - 1, {i, 0, n1 - 1}]
y1 := f[x1]
M = Table[x1[[i + 1]] ^ j, {i, 0, n1 - 1}, {j, 0, n1 - 1}];
M[[(n1 + 1) / 2, 1]] = 1;
A = LinearSolve[M, y1];
```

Power::indet : Indeterminate expression 0^0 encountered.

In[516]:=
```
f1[z_] := Sum[A[[i]] * z ^ (i - 1), {i, 1, n1}]
```

In[517]:=
```
plot1 = Plot[f1[z], {z, -1, 1}];
```

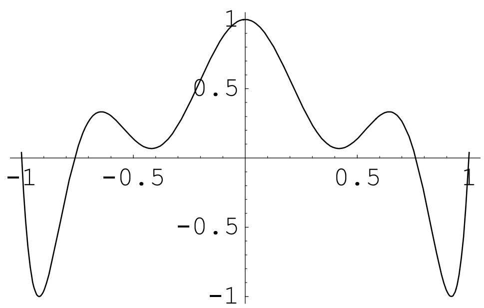


In[518]:=
```
x2 := Table[2 / (n2 - 1) * i - 1, {i, 0, n2 - 1}]
y2 := f[x2]
M = Table[x2[[i + 1]] ^ j, {i, 0, n2 - 1}, {j, 0, n2 - 1}];
M[[(n2 + 1) / 2, 1]] = 1;
A = LinearSolve[M, y2];
```

Power::indet : Indeterminate expression 0^0 encountered.

In[523]:=
```
f2[z_] := Sum[A[[i]] * z^(i - 1), {i, 1, n2}]
```

In[524]:=
```
plot2 = Plot[f2[z], {z, -1, 1}];
```

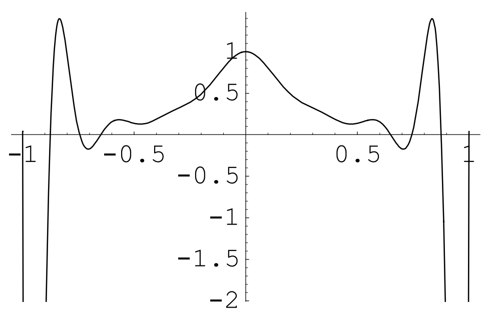


In[525]:=
```
x3 := Table[2 / (n3 - 1) * i - 1, {i, 0, n3 - 1}]
y3 := f[x3]
M = Table[x3[[i + 1]] ^ j, {i, 0, n3 - 1}, {j, 0, n3 - 1}];
M[[(n3 + 1) / 2, 1]] = 1;
A = LinearSolve[M, y3];
```

Power::indet : Indeterminate expression 0^0 encountered.

In[530]:=
```
f3[z_] := Sum[A[[i]] * z^(i - 1), {i, 1, n3}]
```

In[531]:=
```
plot3 = Plot[f3[z], {z, -1, 1}];
```



In[533]:= **Show[Runge, plot1, plot2, plot3];**