# Nonlinear Regression Models without Data Linearization

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```
ClearAll;
Off[General::spell1]
```

## Introduction

This worksheet illustrates finding the constants of a nonlinear regression model without data linearization. Three common nonlinear models are illustrated.

**Exponential:** $y = ae^{bx}$
**Power:** $y = ax^b$
**Saturation:** $y = \frac{(ax)}{b+x}$

where $a$ and $b$ are the constants of the above regression models.

Given $n$ data points $(x_1, y_1)$, $(x_2, y_2)$, $(x_3, y_3)$,...., $(x_n, y_n)$, you can best fit one of the above models to the data. In this worksheet, the constants $a$ and $b$ are calculated in the following steps:

1) Find the sum of the squares of the residuals, $Sr$.
2) Minimize $Sr$ by differentiating with respect to $a$ and $b$ and setting the resulting two equations equal to zero.
3) Solve the two nonlinear equations simultaneously for $a$ and $b$.

*Mathematica* will then return the real solutions of $a$ and $b$. To learn more about nonlinear regression models see the Nonlinear Regression model worksheet.

# Section 1: Input Data

Below are the input parameters to begin the simulation. This is the only section that requires user input. Once the values are entered, *Mathematica* will return the constants of the nonlinear regression model that is specified by the user with the *modeltype* variable.
**NOTE: Before evaluating the worksheet, the user must enter initial guesses of the constants of the model *a* and *b*. For reasonable initial guesses, use the solution from the** Nonlinear Model with data linearization **worksheet. For convergence, use initial guesses *a* and *b* close to the values of *a* and *b* obtained by using data linearization.**

**Input Parameters:**

- Number of data points, *n*

```
n = 5
```

```
5
```

- Array of x values, X

```
X = {10, 16, 25, 40, 60}
```

```
{10, 16, 25, 40, 60}
```

- Array of y values, Y

```
Y = {94, 118, 147, 180, 230}
```

```
{94, 118, 147, 180, 230}
```

- For *exponential* model call model type to be "exponential"
  For *power* model assign the model type variable as "power"
  For *saturation growth* model, assign the model type variable to be "growth"

```
modeltype = "power"
```

```
power
```

- Insert your initial guesses for *a* and *b* here. Reasonable initial guesses for *a* and *b* can be obtained from data linearization models.

Initial Guess values for *a*

```
Ainit = 30.199
```

```
30.199
```

Initial Guess values for *b*

```
Binit = 0.489
```

```
0.489
```

```
Clear[a];
Clear[b];
Clear[Sr];
```

# Section 2: Finding the constants of the model

Assigning the proper regression model

```
If[modeltype == "exponential", f[x_] = a * E^(b * x)];
If[modeltype == "power", f[x_] = a * x^b];
If[modeltype == "growth", f[x_] = (a * x) / (b + x)];
```

Calculating the sum of the square of the residuals

```
Sr = 0;
For[i = 1, i ≤ n, i++, Sr = Sr + ((Y[[i]] - f[X[[i]]])^2)];
```

```
Sr
```

$$(94 - 10^b a)^2 + (118 - 16^b a)^2 + (147 - 25^b a)^2 + (180 - 40^b a)^2 + (230 - 60^b a)^2$$

Differentiating the sum of the square of the residuals with respect to the constants of the model, *a* and *b*, to setup two simultaneous nonlinear equations and two unknowns.

```
eqn1 = Expand[D[Sr, a] == 0];
eqn2 = Expand[D[Sr, b] == 0];
```

Solving the two simultaneous nonlinear equations. We are using *FindRoot* since we are looking for real solutions.

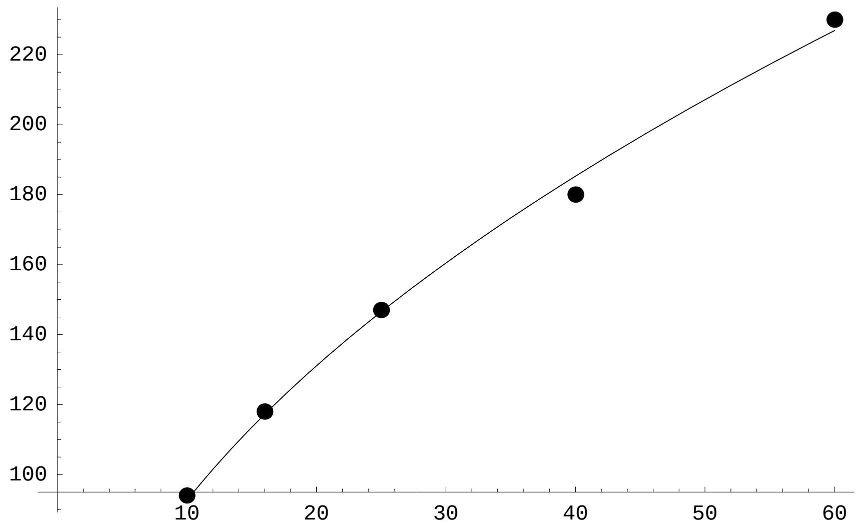
```
soln = FindRoot[{eqn1, eqn2}, {{a, Ainit}, {b, Binit}}, MaxIterations → 1000];
a /. soln[[1]];
a = %;
b /. soln[[2]];
b = %;
Print["The constants of the ", modeltype, " model are a = ", a, " and b = ", b]
```

```
The constants of the power model are a = 29.335 and b = 0.499663
```

Plotting the observed values and the predicted curve

```
predicted = f[x];
observed = Table[{X[[i]], Y[[i]]}, {i, 1, n}];
ttl = Print[modeltype, " regression model, y vs x"];
points = ListPlot[observed, PlotStyle -> PointSize[.02], DisplayFunction → Identity];
lin = Plot[predicted, {x, Min[X], Max[X]}, DisplayFunction → Identity];
Show[points, lin, DisplayFunction → $DisplayFunction]
```

power regression model, y vs x




- Graphics -

## Conclusion

*Mathematica* helped us in illustrating three common nonlinear models of regression. Answer the following questions using the worksheet

1) Verify each of the models by using data that exactly follows the regression model.
2.) What is the difference in the solution between nonlinear models obtained via data that is linearized and data that is not linearized.

## References

[1] Autar Kaw, *Holistic Numerical Methods Institute, http://numericalmethods.eng.usf.edu/nbm,* See How does Nonlinear Regression Work?