07.02
Trapezoidal Rule for Integration-More Examples
Chemical Engineering

Example 1

In an attempt to understand the mechanism of the depolarization process in a fuel cell, an electro-kinetic model for mixed oxygen-methanol current on platinum was developed in the laboratory at FAMU. A very simplified model of the reaction developed suggests a functional relation in an integral form. To find the time required for 50% of the oxygen to be consumed, the time, $T(s)$ is given by

$$T = -\int_{1.22\times10^{-6}}^{0.61\times10^{-6}}\left(\frac{6.73x + 4.3025\times10^{-7}}{2.316\times10^{-11}x}\right)dx$$

a) Use single segment Trapezoidal rule to find the time required for 50% of the oxygen to be consumed.
b) Find the true error, E_t, for part (a).
c) Find the absolute relative true error, $|\in_t|$, for part (a).

Solution

a) $I \approx (b-a)\left[\frac{f(a)+f(b)}{2}\right]$, where

$a = 1.22\times10^{-6}$

$b = 0.61\times10^{-6}$

$$f(x) = -\left[\frac{6.73x + 4.3025\times10^{-7}}{2.316\times10^{-11}x}\right]$$

$$f\left(1.22\times10^{-6}\right) = -\left[\frac{6.73\left(1.22\times10^{-6}\right)+4.3025\times10^{-7}}{2.316\times10^{-11}\left(1.22\times10^{-6}\right)}\right] = -3.0581\times10^{11}$$

$$f\left(0.61\times10^{-6}\right) = -\left[\frac{6.73\left(0.61\times10^{-6}\right)+4.3025\times10^{-7}}{2.316\times10^{-11}\left(0.61\times10^{-6}\right)}\right] = -3.2104\times10^{11}$$

$$I = \left(0.61\times10^{-6} - 1.22\times10^{-6}\right)\left[\frac{-3.0582\times10^{11}+\left(-3.2104\times10^{11}\right)}{2}\right]$$

$$= 1.9119\times10^{5}\,\text{s}$$

b) The exact value of the above integral is,

$$T = -\int_{1.22\times10^{-6}}^{0.61\times10^{-6}} \left(\frac{6.73x + 4.3025\times10^{-7}}{2.316\times10^{-11}x} \right) dx$$

$$= 1.9014\times10^{5}\,\text{s}$$

so the true error is

$$E_t = True\ Value - Approximate\ Value$$
$$= 1.9014\times10^{5} - 1.9119\times10^{5}$$
$$= -1056.2$$

c) The absolute relative true error, $|\in_t|$, would then be

$$|\in_t| = \left| \frac{\text{True Error}}{\text{True Value}} \right| \times 100$$
$$= \left| \frac{-1056.2}{1.9014\times10^{5}} \right| \times 100$$
$$= 0.55549\,\%$$

Example 2

In an attempt to understand the mechanism of the depolarization process in a fuel cell, an electro-kinetic model for mixed oxygen-methanol current on platinum was developed in the laboratory at FAMU. A very simplified model of the reaction developed suggests a functional relation in an integral form. To find the time required for 50% of the oxygen to be consumed, the time, $T(s)$ is given by

$$T = -\int_{1.22\times10^{-6}}^{0.61\times10^{-6}} \left(\frac{6.73x + 4.3025\times10^{-7}}{2.316\times10^{-11}x} \right) dx$$

a) Use two- segment Trapezoidal rule to find the time required for 50% of the oxygen to be consumed.
b) Find the true error, E_t, for part (a).
c) Find the absolute relative true error, $|\in_t|$, for part (a).

Solution

a) $$I = \frac{b-a}{2n}\left[f(a) + 2\left\{ \sum_{i=1}^{n-1} f(a+ih) \right\} + f(b) \right]$$

$$n = 2$$
$$a = 1.22\times10^{-6}$$
$$b = 0.61\times10^{-6}$$
$$f(x) = -\left[\frac{6.73x + 4.3025\times10^{-7}}{2.316\times10^{-11}x} \right]$$

$$h = \frac{b-a}{n}$$
$$= \frac{0.61\times10^{-6} - 1.22\times10^{-6}}{2}$$
$$= -0.30500\times10^{-6}$$
$$f(x_0) = f\left(1.22\times10^{-6}\right)$$
$$f\left(1.22\times10^{-6}\right) = -\left[\frac{6.73\left(1.22\times10^{-6}\right)+4.3025\times10^{-7}}{2.316\times10^{-11}\left(1.22\times10^{-6}\right)}\right] = -3.0581\times10^{11}$$
$$f(x_1) = f\left(1.22\times10^{-6} - 0.30500\times10^{-6}\right)$$
$$= f\left(0.91500\times10^{-6}\right)$$
$$f(0.91500) = -\left[\frac{6.73\left(0.915\times10^{-6}\right)+4.3025\times10^{-7}}{2.316\times10^{-11}\left(0.915\times10^{-6}\right)}\right] = -3.1089\times10^{11}$$
$$f(x_2) = f(x_n) = f\left(0.61\times10^{-6}\right)$$
$$f\left(0.61\times10^{-6}\right) = -\left[\frac{6.73\left(0.61\times10^{-6}\right)+4.3025\times10^{-7}}{2.316\times10^{-11}\left(0.61\times10^{-6}\right)}\right] = -3.2104\times10^{11}$$
$$I = \frac{0.61\times10^{-6} - 1.22\times10^{-6}}{2(2)}\left[f\left(1.22\times10^{-6}\right) + 2\left\{\sum_{i=1}^{2-1} f(a+ih)\right\} + f\left(0.61\times10^{-6}\right)\right]$$
$$= \frac{-0.61\times10^{-6}}{4}\left[f\left(1.22\times10^{-6}\right) + 2f\left(0.915\times10^{-6}\right) + f\left(0.61\times10^{-6}\right)\right]$$
$$= \frac{-0.61\times10^{-6}}{4}\left[-3.0581\times10^{11} + 2\left(-3.1089\times10^{11}\right) - 3.2104\times10^{11}\right]$$
$$= 1.9042\times10^{5}\ \text{s}$$

b) The exact value of the above integral is,

$$T = -\int_{1.22\times10^{-6}}^{0.61\times10^{-6}}\left(\frac{6.73x+4.3025\times10^{-7}}{2.316\times10^{-11}x}\right)dx$$
$$= 1.90140\times10^{5}\ \text{s}$$

so the true error is

$$E_t = True\ Value - Approximate\ Value$$
$$= 1.90140\times10^{5} - 1.9042\times10^{5}$$
$$= -282.12$$

c) The absolute relative true error, $|\in_t|$, would then be

$$|\in_t| = \left|\frac{\text{True Error}}{\text{True Value}}\right|\times100$$
$$= \left|\frac{-282.12}{1.9014\times10^{5}}\right|\times100$$
$$= 0.14838\,\%$$

Table 1 Values obtained using multiple-segment Trapezoidal rule for

$$T = -\int_{1.22\times10^{-6}}^{0.61\times10^{-6}} \left(\frac{6.73x + 4.3025\times10^{-7}}{2.316\times10^{-11}x} \right) dx$$

n	Value	E_t	$\lvert \in_t \rvert \%$	$\lvert \in_a \rvert \%$
1	191190	−1056.2	0.55549	---
2	190420	−282.12	0.14838	0.40711
3	190260	−127.31	0.066956	0.081424
4	190210	−72.017	0.037877	0.029079
5	190180	−46.216	0.024307	0.013570
6	190170	−32.142	0.016905	0.0074020
7	190160	−23.636	0.012431	0.0044740
8	190150	−18.107	0.0095231	0.0029079