Chapter 07.02
Trapezoidal Rule for Integration-More Examples
Civil Engineering

Example 1

The concentration of benzene at a critical location is given by

$$c = 1.75\left[erfc(0.6560) + e^{32.73} erfc(5.758)\right]$$

where

$$erfc(x) = \int_{\infty}^{x} e^{-z^2} dz$$

So in the above formula

$$erfc(0.6560) = \int_{\infty}^{0.6560} e^{-z^2} dz$$

Since e^{-z^2} decays rapidly as $z \to \infty$, we will approximate

$$erfc(0.6560) = \int_{5}^{0.6560} e^{-z^2} dz$$

a) Use single segment Trapezoidal rule to find the value of $erfc(0.6560)$.
b) Find the true error, E_t, for part (a).
c) Find the absolute relative true error for part (a).

Solution

a) $I \approx (b-a)\left[\dfrac{f(a)+f(b)}{2}\right]$, where

$$a = 5$$
$$b = 0.6560$$
$$f(z) = e^{-z^2}$$
$$f(5) = e^{-5^2}$$
$$= 1.3888 \times 10^{-11}$$
$$f(0.6560) = e^{-0.6560^2}$$
$$= 0.65029$$
$$I = (0.6560 - 5)\left[\frac{1.3888 \times 10^{-11} + 0.65029}{2}\right]$$
$$= -1.4124$$

b) The exact value of the above integral cannot be found. We assume the value obtained by adaptive numerical integration using Maple as the exact value for calculating the true error and relative true error.

$$erfc(0.6560) = \int_{5}^{0.6560} e^{-z^2} dz$$

$$= -0.31333$$

so the true error is

$$E_t = True\ Value - Approximate\ Value$$
$$= -0.31333 - (-1.4124)$$
$$= 1.0991$$

c) The absolute relative true error, $|\in_t|$, would then be

$$|\in_t| = \left|\frac{\text{True Error}}{\text{True Value}}\right| \times 100$$
$$= \left|\frac{-0.31333 - (-1.4124)}{-0.31333}\right| \times 100$$
$$= 350.79\ \%$$

Example 2

The concentration of benzene at a critical location is given by

$$c = 1.75\left[erfc(0.6560) + e^{32.73} erfc(5.758)\right]$$

where

$$erfc(x) = \int_{\infty}^{x} e^{-z^2} dz$$

So in the above formula

$$erfc(0.6560) = \int_{\infty}^{0.6560} e^{-z^2} dz$$

Since e^{-z^2} decays rapidly as $z \to \infty$, we will approximate

$$erfc(0.6560) = \int_{5}^{0.6560} e^{-z^2} dz$$

a) Use two segment Trapezoidal rule to find the value of $erfc(0.6560)$.
b) Find the true error, E_t, for part (a).
c) Find the absolute relative true error for part (a).

Solution

a) The solution using 2-segment Trapezoidal rule is

$$I \approx \frac{b-a}{2n}\left[f(a)+2\left\{\sum_{i=1}^{n-1} f(a+ih)\right\}+f(b)\right]$$

$$n = 2$$
$$a = 5$$
$$b = 0.6560$$
$$h = \frac{b-a}{n}$$
$$= \frac{0.6560-5}{2}$$
$$= -2.172$$

$$I \approx \frac{0.6560-5}{2(2)}\left[f(5)+2\left\{\sum_{i=1}^{2-1} f(a+ih)\right\}+f(0.6560)\right]$$
$$\approx \frac{-4.344}{4}\left[f(5)+2\sum_{i=1}^{1} f(5+i\times(-2.172))+f(0.6560)\right]$$
$$\approx \frac{-4.344}{4}\left[f(5)+2f(5+1\times(-2.172))+f(0.6560)\right]$$
$$\approx \frac{-4.344}{4}\left[f(5)+2f(2.828)+f(0.6560)\right]$$
$$\approx \frac{-4.344}{4}\left[1.3888\times10^{-11}+2(0.00033627)+0.65029\right]$$
$$\approx -0.70695$$

Since

$$f(5) = e^{-5^2}$$
$$= 1.3888\times10^{-11}$$
$$f(2.828) = e^{-2.828^2}$$
$$= 0.00033627$$
$$f(0.6560) = e^{-0.6560^2}$$
$$= 0.65029$$

b) The exact value of the above integral cannot be found. For calculating the true error and relative true error, we assume the value obtained by adaptive numerical integration using Maple as the exact value.

$$erfc(0.6560) = \int_{5}^{0.6560} e^{-z^2}\,dz$$
$$= -0.31333$$

so the true error is

$$E_t = True\ Value - Approximate\ Value$$
$$= -0.31333 - (-0.70695)$$
$$= 0.39362$$

c) The absolute relative true error, $|\in_t|$, would then be

$$|\in_t| = \left|\frac{\text{True Error}}{\text{True Value}}\right| \times 100\,\%$$

$$= \left|\frac{-0.31333 - (-0.70695)}{-0.31333}\right| \times 100\,\%$$
$$= 125.63\,\%$$

Table 1 Values obtained using multiple-segment Trapezoidal rule for
$$erfc(0.6560) = \int_{5}^{0.6560} e^{-z^2}\,dz$$

n	Value	E_t	$\lvert\in_t\rvert$ %	$\lvert\in_a\rvert$ %
1	−1.4124	1.0991	350.79	---
2	−0.70695	0.39362	125.63	99.793
3	−0.48812	0.17479	55.787	44.829
4	−0.40571	0.092379	29.483	20.314
5	−0.37028	0.056957	18.178	9.5662
6	−0.35212	0.038791	12.380	5.1591
7	−0.34151	0.028182	8.9946	3.1063
8	−0.33475	0.021426	6.8383	2.0183