Chapter 05.03
Newton's Divided Difference Interpolation – More Examples
Electrical Engineering

Example 1

Thermistors are used to measure the temperature of bodies. Thermistors are based on materials' change in resistance with temperature. To measure temperature, manufacturers provide you with a temperature vs. resistance calibration curve. If you measure resistance, you can find the temperature. A manufacturer of thermistors makes several observations with a thermistor, which are given in Table 1.

Table 1 Temperature as a function of resistance.

R (ohm)	T (°C)
1101.0	25.113
911.3	30.131
636.0	40.120
451.1	50.128

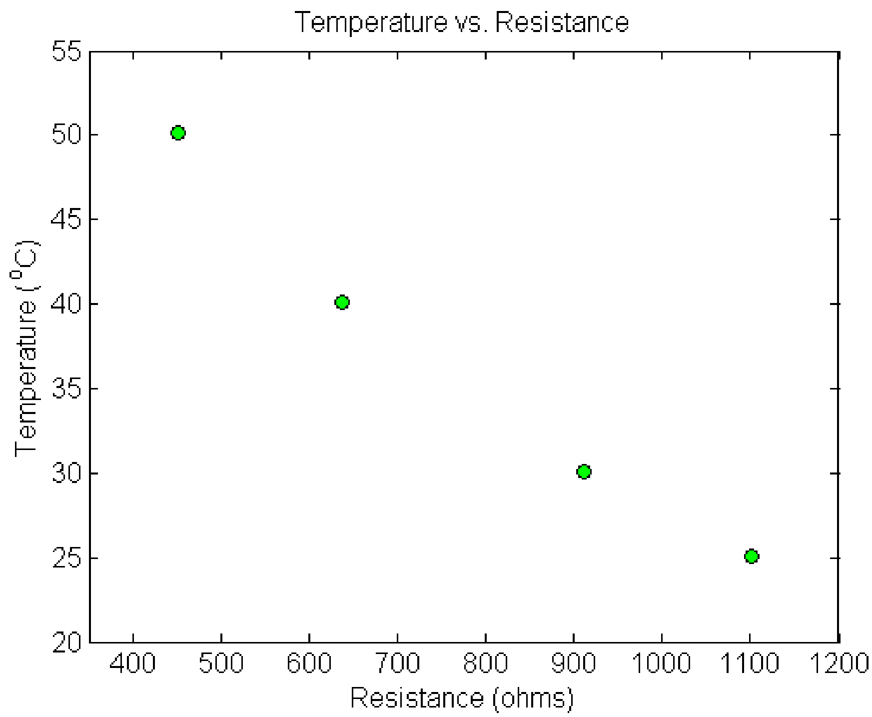

Figure 1 Resistance vs. temperature.

Determine the temperature corresponding to 754.8 ohms using Newton's divided difference method of interpolation and a first order polynomial.

Solution

For linear interpolation, the temperature is given by

$$T(R) = b_0 + b_1(R - R_0)$$

Since we want to find the temperature at $R = 754.8$ and we are using a first order polynomial, we need to choose the two data points that are closest to $R = 754.8$ that also bracket $R = 754.8$ to evaluate it. The two points are $R = 911.3$ and $R = 636.0$.

Then

$$R_0 = 911.3, \; T(R_0) = 30.131$$

$$R_1 = 636.0, \; T(R_1) = 40.120$$

gives

$$b_0 = T(R_0)$$

$$= 30.131$$

$$b_1 = \frac{T(R_1) - T(R_0)}{R_1 - R_0}$$

$$= \frac{40.120 - 30.131}{636.0 - 911.3}$$

$$= -0.036284$$

Hence

$$T(R) = b_0 + b_1(R - R_0)$$
$$= 30.131 - 0.036284(R - 911.3), \qquad 636.0 \le R \le 911.3$$

At $R = 754.8$

$$T(754.8) = 30.131 - 0.036284(754.8 - 911.3)$$
$$= 35.809\,°\text{C}$$

If we expand

$$T(R) = 30.131 - 0.036284(R - 911.3), \qquad 636.0 \le R \le 911.3$$

we get

$$T(R) = 63.197 - 0.036284R, \qquad 636.0 \le R \le 911.3$$

This is the same expression that was obtained with the direct method.

Example 2

Thermistors are used to measure the temperature of bodies. Thermistors are based on materials' change in resistance with temperature. To measure temperature, manufacturers provide you with a temperature vs. resistance calibration curve. If you measure resistance, you can find the temperature. A manufacturer of thermistors makes several observations with a thermistor, which are given in Table 2.

Table 2 Temperature as a function of resistance.

R (ohm)	T (°C)
1101.0	25.113
911.3	30.131
636.0	40.120
451.1	50.128

Determine the temperature corresponding to 754.8 ohms using Newton's divided difference method of interpolation and a second order polynomial. Find the absolute relative approximate error for the second order polynomial approximation.

Solution

For quadratic interpolation, the temperature is given by

$$T(R) = b_0 + b_1(R - R_0) + b_2(R - R_0)(R - R_1)$$

Since we want to find the temperature at $R = 754.8$ and we are using a second order polynomial, we need to choose the three data points that are closest to $R = 754.8$ that also bracket $R = 754.8$ to evaluate it. The three points are $R_0 = 911.3$, $R_1 = 636.0$ and $R_2 = 451.1$.

Then

$$R_0 = 911.3, \quad T(R_0) = 30.131$$
$$R_1 = 636.0, \quad T(R_1) = 40.120$$
$$R_2 = 451.1, \quad T(R_2) = 50.128$$

gives

$$b_0 = T(R_0)$$
$$= 30.131$$
$$b_1 = \frac{T(R_1) - T(R_0)}{R_1 - R_0}$$
$$= \frac{40.120 - 30.131}{636.0 - 911.3}$$
$$= -0.036284$$
$$b_2 = \frac{\dfrac{T(R_2) - T(R_1)}{R_2 - R_1} - \dfrac{T(R_1) - T(R_0)}{R_1 - R_0}}{R_2 - R_0}$$
$$= \frac{\dfrac{50.128 - 40.120}{451.1 - 636.0} - \dfrac{40.120 - 30.131}{636.0 - 911.3}}{451.1 - 911.3}$$
$$= \frac{-0.054127 + 0.036284}{-460.2}$$
$$= 3.8771 \times 10^{-5}$$

Hence

$$T(R) = b_0 + b_1(R - R_0) + b_2(R - R_0)(R - R_1)$$
$$= 30.131 - 0.036284(R - 911.3) + 3.8771 \times 10^{-5}(R - 911.3)(R - 636.0), \quad 451.1 \le R \le 911.3$$

At $R = 754.8$,

$$T(754.8) = 30.131 - 0.036284(754.8 - 911.3) + 3.8771 \times 10^{-5}(754.8 - 911.3)(754.8 - 636.0)$$
$$= 35.089\,°\mathrm{C}$$

The absolute relative approximate error $|\in_a|$ obtained between the results from the first and second order polynomial is

$$|\in_a| = \left|\frac{35.089 - 35.809}{35.089}\right| \times 100$$
$$= 2.0543\%$$

If we expand,

$$T(R) = 30.131 - 0.036284(R - 911.3) + 3.8771 \times 10^{-5}(R - 911.3)(R - 636.0), \quad 451.1 \le R \le 911.3$$

we get

$$T(R) = 85.668 - 0.096275R + 3.8771 \times 10^{-5} R^2, \quad 451.1 \le R \le 911.3$$

This is the same expression that was obtained with the direct method.

Example 3

Thermistors are used to measure the temperature of bodies. Thermistors are based on materials' change in resistance with temperature. To measure temperature, manufacturers provide you with a temperature vs. resistance calibration curve. If you measure resistance, you can find the temperature. A manufacturer of thermistors makes several observations with a thermistor, which are given in Table 3.

Table 3 Temperature as a function of resistance.

R (ohm)	T (°C)
1101.0	25.113
911.3	30.131
636.0	40.120
451.1	50.128

a) Determine the temperature corresponding to 754.8 ohms using Newton's divided difference method of interpolation and a third order polynomial. Find the absolute relative approximate error for the third order polynomial approximation.
b) The actual calibration curve used by industry is given by

$$\frac{1}{T} = b_0 + b_1(\ln R - \ln R_0) + b_2(\ln R - \ln R_0)(\ln R - \ln R_1) + b_3(\ln R - \ln R_0)(\ln R - \ln R_1)(\ln R - \ln R_2)$$

substituting $y = \frac{1}{T}$, and $x = \ln R$,

the calibration curve is given by

$$y(x) = b_0 + b_1(x - x_0) + b_2(x - x_0)(x - x_1) + b_3(x - x_0)(x - x_1)(x - x_2)$$

Table 4 Manipulation for the given data.

R (ohm)	T (°C)	x ($\ln R$)	$y\left(\frac{1}{T}\right)$
1101.0	25.113	7.0040	0.039820
911.3	30.131	6.8149	0.033188
636.0	40.120	6.4552	0.024925
451.1	50.128	6.1117	0.019949

Find the calibration curve and find the temperature corresponding to 754.8 ohms. What is the difference between the results from part (a)? Is the difference larger using results from part (a) or part (b), if the actual measured value at 754.8 ohms is $35.285\,°\mathrm{C}$?

Solution

a) For cubic interpolation, the temperature is given by

$$T(R) = b_0 + b_1(R - R_0) + b_2(R - R_0)(R - R_1) + b_3(R - R_0)(R - R_1)(R - R_2)$$

Since we want to find the temperature at $R = 754.8$, we need to choose the four data points that are closest to $R = 754.8$ that also bracket $R = 754.8$ to evaluate it. The four data points are $R_0 = 1101.0$, $R_1 = 911.3$, $R_2 = 636.0$ and $R_3 = 451.1$.

Then

$$R_0 = 1101.0,\ T(R_0) = 25.113$$

$$R_1 = 911.3, \; T(R_1) = 30.131$$
$$R_2 = 636.0, \; T(R_2) = 40.120$$
$$R_3 = 451.1, \; T(R_3) = 50.128$$

gives

$$b_0 = T[R_0]$$
$$= T(R_0)$$
$$= 25.113$$
$$b_1 = T[R_1, R_0]$$
$$= \frac{T(R_1) - T(R_0)}{R_1 - R_0}$$
$$= \frac{30.131 - 25.113}{911.3 - 1101.0}$$
$$= -0.026452$$
$$b_2 = T[R_2, R_1, R_0]$$
$$= \frac{T[R_2, R_1] - T[R_1, R_0]}{R_2 - R_0}$$
$$T[R_2, R_1] = \frac{T(R_2) - T(R_1)}{R_2 - R_1}$$
$$= \frac{40.120 - 30.131}{636.0 - 911.3}$$
$$= -0.036284$$
$$T[R_1, R_0] = -0.026452$$
$$b_2 = \frac{T[R_2, R_1] - T[R_1, R_0]}{R_2 - R_0}$$
$$= \frac{-0.036284 + 0.026452}{636.0 - 1101.0}$$
$$= 2.1144 \times 10^{-5}$$

$$b_3 = T[R_3, R_2, R_1, R_0]$$
$$= \frac{T[R_3, R_2, R_1] - T[R_2, R_1, R_0]}{R_3 - R_0}$$
$$T[R_3, R_2, R_1] = \frac{T[R_3, R_2] - T[R_2, R_1]}{R_3 - R_1}$$
$$T[R_3, R_2] = \frac{T(R_3) - T(R_2)}{R_3 - R_2}$$
$$= \frac{50.128 - 40.120}{451.1 - 636.0}$$
$$= -0.054127$$
$$T[R_2, R_1] = -0.036284$$

$$T[R_3,R_2,R_1] = \frac{T[R_3,R_2]-T[R_2,R_1]}{R_3-R_1}$$
$$= \frac{-0.054127+0.036284}{451.1-911.3}$$
$$= 3.8771\times10^{-5}$$
$$T[R_2,R_1,R_0] = 2.1144\times10^{-5}$$
$$b_3 = \frac{T[R_3,R_2,R_1]-T[R_2,R_1,R_0]}{R_3-R_0}$$
$$= \frac{3.8771\times10^{-5}-2.1144\times10^{-5}}{451.1-1101.0}$$
$$= -2.7124\times10^{-8}$$

Hence

$$T(R) = b_0 + b_1(R-R_0) + b_2(R-R_0)(R-R_1) + b_3(R-R_0)(R-R_1)(R-R_2)$$
$$= 25.113 - 0.026452(R-1101.0) + 2.1144\times10^{-5}(R-1101.0)(R-911.3)$$
$$-2.7124\times10^{-8}(R-1101.0)(R-911.3)(R-636.0),\ \ 451.1\le R\le 1101.0$$

At $R = 754.8$,

$$T(754.8) = 25.113 - 0.026452(754.8-1101.0) + 2.1144\times10^{-5}(754.8-1101.0)(754.8-911.3)$$
$$-2.7124\times10^{-8}(754.8-1101.0)(754.8-911.3)(754.8-626.0)$$
$$= 35.242\,°\text{C}$$

The absolute relative approximate error $|\in_a|$ obtained between the results from the second and third order polynomial is

$$|\in_a| = \left|\frac{35.242-35.089}{35.242}\right|\times100$$
$$= 0.43458\%$$

If we expand

$$T(R) = 25.113 - 0.026452(R-1101.0) + 2.1144\times10^{-5}(R-1101.0)(R-911.3)$$
$$-2.7124\times10^{-8}(R-1101.0)(R-911.3)(R-636.0),\qquad 451.1\le R\le 1101.0$$

we get

$$T(R) = 92.759 - 0.13093R + 9.2975\times10^{-5}R^2 - 2.7124\times10^{-8}R^3,\ 451.1\le R\le 1101.0$$

This is the same expression that was obtained with the direct method.

b) Finding the cubic interpolant using Newton's divided difference for

$$y(x) = b_0 + b_1(x-x_0) + b_2(x-x_0)(x-x_1) + b_3(x-x_0)(x-x_1)(x-x_2)$$

requires that we first calculate the new values of x and y.

$x\ (\ln R)$	$y\ \left(\frac{1}{T}\right)$
7.0040	0.039820
6.8149	0.033188
6.4552	0.024925
6.1117	0.019949

Then

$$x_0 = 7.0040,\quad y(x_0) = 0.039820$$
$$x_1 = 6.8149,\quad y(x_1) = 0.033188$$
$$x_2 = 6.4552,\quad y(x_2) = 0.024925$$
$$x_3 = 6.1117,\quad y(x_3) = 0.019949$$

gives

$$\begin{aligned}b_0 &= y[x_0] \\ &= y(x_0) \\ &= 0.039820\end{aligned}$$

$$\begin{aligned}b_1 &= y[x_1, x_0] \\ &= \frac{y(x_1) - y(x_0)}{x_1 - x_0} \\ &= \frac{0.033188 - 0.039820}{6.8149 - 7.0040} \\ &= 0.035069\end{aligned}$$

$$\begin{aligned}b_2 &= y[x_2, x_1, x_0] \\ &= \frac{y[x_2, x_1] - y[x_1, x_0]}{x_2 - x_0}\end{aligned}$$

$$\begin{aligned}y[x_2, x_1] &= \frac{y(x_2) - y(x_1)}{x_2 - x_1} \\ &= \frac{0.024925 - 0.033188}{6.4552 - 6.8149} \\ &= 0.022974\end{aligned}$$

$$y[x_1, x_0] = 0.035069$$

$$\begin{aligned}b_2 &= \frac{y[x_2, x_1] - y[x_1, x_0]}{x_2 - x_0} \\ &= \frac{0.022974 - 0.035069}{6.4552 - 7.0040} \\ &= 0.022040\end{aligned}$$

$$b_3 = y[x_3, x_2, x_1, x_0]$$

$$= \frac{y[x_3,x_2,x_1]-y[x_2,x_1,x_0]}{x_3-x_0}$$

$$y[x_3,x_2,x_1] = \frac{y[x_3,x_2]-y[x_2,x_1]}{x_3-x_1}$$

$$y[x_3,x_2] = \frac{y(x_3)-y(x_2)}{x_3-x_2}$$

$$= \frac{0.019949-0.024925}{6.1117-6.4552}$$

$$= 0.014487$$

$$y[x_2,x_1] = 0.022974$$

$$y[x_3,x_2,x_1] = \frac{y[x_3,x_2]-y[x_2,x_1]}{x_3-x_1}$$

$$= \frac{0.014487-0.022974}{6.1117-6.8149}$$

$$= 0.012070$$

$$y[x_2,x_1,x_0] = 0.022040$$

$$b_3 = \frac{y[x_3,x_2,x_1]-y[x_2,x_1,x_0]}{x_3-x_0}$$

$$= \frac{0.012070-0.022040}{6.1117-7.0040}$$

$$= 0.011173$$

Hence

$$y(x) = b_0 + b_1(x-x_0) + b_2(x-x_0)(x-x_1) + b_3(x-x_0)(x-x_1)(x-x_2)$$

$$= 0.039820 + 0.035069(x-7.0040) + 0.022974(x-7.0040)(x-6.8149)$$

$$+ 0.011173(x-7.0040)(x-6.8149)(x-6.4552), \quad 6.1117 \le x \le 7.0040$$

Since we're looking for the temperature at $R = 754.8$, we will be using

$$x = \ln(754.8)$$

$$= 6.6265$$

At $x = 6.6265$,

$$y(6.6265) = 0.039820 + 0.035071(6.6265-7.0040)$$

$$+ 0.022972(6.6265-7.0040)(6.6265-6.8149)$$

$$+ 0.011182(6.6265-7.0040)(6.6265-6.8149)(6.6265-6.4552)$$

$$= 0.028285$$

Finally, since $y = \frac{1}{T}$,

$$T = \frac{1}{y}$$

$$= \frac{1}{0.028285}$$
$$= 35.355°\text{C}$$

Since the actual measured value at 754.8 ohms is $35.285°\text{C}$, the absolute relative true error for the value found in part (a) is

$$|\in_t| = \left|\frac{35.285 - 35.242}{35.285}\right| \times 100$$
$$= 0.12253\%$$

and for part (b) is

$$|\in_t| = \left|\frac{35.285 - 35.355}{35.285}\right| \times 100$$
$$= 0.19825\%$$

Therefore, the cubic polynomial interpolant given by Newton's divided difference method, that is,

$$T(R) = b_0 + b_1(R - R_0) + b_2(R - R_0)(R - R_1) + b_3(R - R_0)(R - R_1)(R - R_2)$$

obtained more accurate results than the calibration curve of

$$\frac{1}{T} = b_0 + b_1(\ln R - \ln R_0) + b_2(\ln R - \ln R_0)(\ln R - \ln R_1) + b_3(\ln R - \ln R_0)(\ln R - \ln R_1)(\ln R - \ln R_2)$$

INTERPOLATION	
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