Chapter 03.04
Newton-Raphson Method of Solving a Nonlinear Equation – More Examples
Industrial Engineering

Example 1

You are working for a start-up computer assembly company and have been asked to determine the minimum number of computers that the shop will have to sell to make a profit. The equation that gives the minimum number of computers n to be sold after considering the total costs and the total sales is

$$f(n) = 40n^{1.5} - 875n + 35000 = 0$$

Use the Newton-Raphson method of finding roots of equations to find the minimum number of computers that need to be sold to make a profit. Conduct three iterations to estimate the root of the above equations. Find the absolute relative approximate error at the end of each iteration and the number of significant digits at least correct at the end of each iteration.

Solution

$$f(n) = 40n^{1.5} - 875n + 35000 = 0$$
$$f'(n) = 60n^{0.5} - 875$$

Let us take the initial guess of the root of $f(n) = 0$ as $n_0 = 50$.

Iteration 1
The estimate of the root is

$$n_1 = n_0 - \frac{f(n_0)}{f'(n_0)}$$

$$= 50 - \frac{40(50)^{1.5} - 875(50) + 35000}{60(50)^{0.5} - 875}$$

$$= 50 - \frac{5392.1}{-450.74}$$

$$= 50 - (-11.963)$$

$$= 61.963$$

The absolute relative approximate error $\left| \in_a \right|$ at the end of Iteration 1 is

$$|\epsilon_a| = \left|\frac{n_1 - n_0}{n_1}\right| \times 100$$

$$= \left|\frac{61.963 - 50}{61.963}\right| \times 100$$

$$= 19.307\%$$

The number of significant digits at least correct is 0, as you need an absolute relative approximate error of less than 5% for one significant digit to be correct in your result.

Iteration 2
The estimate of the root is

$$n_2 = n_1 - \frac{f(n_1)}{f'(n_1)}$$

$$= 61.963 - \frac{40(61.963)^{1.5} - 875(61.963) + 35000}{60(61.963)^{0.5} - 875}$$

$$= 61.963 - \frac{292.45}{-402.70}$$

$$= 61.963 - (-0.72623)$$

$$= 62.689$$

The absolute relative approximate error $|\epsilon_a|$ at the end of Iteration 2 is

$$|\epsilon_a| = \left|\frac{n_2 - n_1}{n_2}\right| \times 100$$

$$= \left|\frac{62.689 - 61.963}{62.689}\right| \times 100$$

$$= 1.1585\%$$

The number of significant digits at least correct is 1, because the absolute relative approximate error is less than 5%.

Iteration 3
The estimate of the root is

$$n_3 = n_2 - \frac{f(n_2)}{f'(n_2)}$$

$$= 62.689 - \frac{40(62.689)^{1.5} - 875(62.689) + 35000}{60(62.689)^{0.5} - 875}$$

$$= 62.689 - \frac{1.0031}{-399.94}$$

$$= 62.689 - (-2.5080 \times 10^{-3})$$

$$= 62.692$$

The absolute relative approximate error $|\epsilon_a|$ at the end of Iteration 3 is

$$|\epsilon_a| = \left|\frac{n_3 - n_2}{n_3}\right| \times 100$$

$$|\epsilon_a| = \left|\frac{62.692 - 62.689}{62.692}\right| \times 100$$

$$= 4.0006 \times 10^{-3}\%$$

Hence the number of significant digits at least correct is given by the largest value of m for which

$$|\epsilon_a| \le 0.5 \times 10^{2-m}$$

$$4.0006 \times 10^{-3} \le 0.5 \times 10^{2-m}$$

$$8.0011 \times 10^{-3} \le 10^{2-m}$$

$$\log\left(8.0011 \times 10^{-3}\right) \le 2 - m$$

$$m \le 2 - \log\left(8.0011 \times 10^{-3}\right) = 4.0968$$

So

$$m = 4$$

The number of significant digits at least correct in the estimated root 62.692 is 4.

NONLINEAR EQUATIONS	
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