07.03
Simpson's 1/3 Rule for Integration-More Examples Industrial Engineering

Example 1

A company advertises that every roll of toilet paper has at least 250 sheets. The probability that there are 250 or more sheets in the toilet paper is given by

$$P(y \geq 250) = \int_{250}^{\infty} 0.3515 \ e^{-0.3881(y-252.2)^2} dy$$

Approximating the above integral as

$$P(y \geq 250) = \int_{250}^{270} 0.3515 \ e^{-0.3881(y-252.2)^2} dy$$

a) Use Simpson's 1/3 Rule to find the probability.
b) Find the true error, E_t, for part (a).
c) Find the absolute relative true error, $|\in_t|$, for part (a).

Solution

a) $$P(y \geq 250) \approx \frac{b-a}{6}\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right]$$

$$a = 250$$
$$b = 270$$
$$\frac{a+b}{2} = 260$$
$$f(y) = 0.3515e^{-0.3881(y-252.2)^2}$$
$$f(250) = 0.3515e^{-0.3881(250-252.2)^2}$$
$$= 0.053721$$
$$f(270) = 0.3515e^{-0.3881(270-252.2)^2}$$
$$= 1.3888 \times 10^{-54}$$
$$f(260) = 0.3515e^{-0.3881(260-252.2)^2}$$
$$= 1.9560 \times 10^{-11}$$
$$P(y \geq 250) \approx \left(\frac{b-a}{6}\right)\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right]$$
$$\approx \left(\frac{270-250}{6}\right)[f(250) + 4f(260) + f(270)]$$

$$\approx \left(\frac{20}{6}\right)\left[0.053721 + 4\left(1.9559\times 10^{-11}\right) + 1.3888\times 10^{-54}\right]$$
$$\approx 0.17907$$

b) The exact value of the above integral cannot be found. For calculating the true error and relative true error, we assume the value obtained by adaptive numerical integration using Maple as the exact value.

$$P(y \geq 250) = \int_{250}^{270} 0.3515\ e^{-0.3881(y-252.2)^2} dy$$
$$= 0.97377$$

So the true error is,

$$E_t = \text{True Value} - \text{Approximate Value}$$
$$= 0.97377 - 0.17907$$
$$= 0.79470$$

Absolute Relative true error,

$$\left|\in_t\right| = \left|\frac{\text{True Error}}{\text{True Value}}\right| \times 100\,\%$$
$$= \left|\frac{0.79470}{0.97377}\right| \times 100\,\%$$
$$= 81.611\,\%$$

Example 2

A company advertises that every roll of toilet paper has at least 250 sheets. The probability that there are 250 or more sheets in the toilet paper is given by

$$P(y \geq 250) = \int_{250}^{\infty} 0.3515\ e^{-0.3881(y-252.2)^2} dy$$

Approximating the above integral as

$$P(y \geq 250) = \int_{250}^{270} 0.3515\ e^{-0.3881(y-252.2)^2} dy$$

a) Use four segment Simpson's 1/3 Rule to find the probability.
b) Find the true error, E_t, for part (a).
c) Find the absolute relative true error for part (a).

Solution

a) Using n segment Simpson's 1/3rd Rule,

$$P(y \geq 250) \approx \frac{b-a}{3n}\left[f(y_0) + 4\sum_{\substack{i=1\\ i=odd}}^{n-1} f(y_i) + 2\sum_{\substack{i=2\\ i=even}}^{n-2} f(y_i) + f(y_n)\right]$$

$$n = 4$$
$$a = 250$$
$$b = 270$$
$$h = \frac{b-a}{n}$$

$$= \frac{270-250}{4}$$
$$= 5$$

$$f(y) = 0.3515e^{-0.3881(y-252.2)^2}$$

So

$$P(y \geq 250) \approx \frac{b-a}{3n}\left[f(y_0) + 4\sum_{\substack{i=1\\ i=odd}}^{n-1} f(y_i) + 2\sum_{\substack{i=2\\ i=even}}^{n-2} f(y_i) + f(y_n)\right]$$

$$\approx \frac{270-250}{3(4)}\left[f(250) + 4\sum_{\substack{i=1\\ i=odd}}^{3} f(y_i) + 2\sum_{\substack{i=2\\ i=even}}^{2} f(y_i) + f(270)\right]$$

$$\approx \frac{20}{12}\left[f(250) + 4f(y_1) + 4f(y_3) + 2f(y_2) + f(270)\right]$$

$$\approx \frac{10}{6}\left[f(250) + 4f(255) + 4f(265) + 2f(260) + f(270)\right]$$

$$\approx \frac{10}{6}\left[\begin{array}{l}0.053721 + 4(0.016769) + 4(8.5260\times 10^{-29})\\ + 2(1.9560\times 10^{-11}) + 1.3888\times 10^{-54}\end{array}\right]$$

$$\approx 0.20133$$

Since

$$f(y_0) = f(250)$$
$$= 0.3515e^{-0.3881(250-252.2)^2}$$
$$= 0.053721$$

$$f(y_1) = f(250+5)$$
$$= f(255)$$
$$= 0.3515e^{-0.3881(255-252.2)^2}$$
$$= 0.016769$$

$$f(y_2) = f(255+5)$$
$$= f(260)$$
$$= 0.3515e^{-0.3881(260-252.2)^2}$$
$$= 1.9560\times 10^{-11}$$

$$f(y_3) = f(260+5)$$
$$= f(265)$$
$$= 0.3515e^{-0.3881(265-252.2)^2}$$
$$= 8.5260\times 10^{-29}$$

$$f(y_4) = f(y_n)$$

$$= f(270)$$
$$= 0.3515e^{-0.3881(270-252.2)^2}$$
$$= 1.3888 \times 10^{-54}$$

b) The exact value of the above integral cannot be found. For calculating the true error and relative true error, we assume the value obtained by adaptive numerical integration using Maple as the exact value.

$$P(y \geq 250) = \int_{250}^{270} 0.3515 \ e^{-0.3881(y-252.2)^2} dy$$
$$= 0.97377$$

So the true error is

$$E_t = True\ Value - Approximate\ Value$$
$$= 0.97377 - 0.20133$$
$$= 0.77244$$

Absolute Relative true error,

$$|\in_t| = \left|\frac{\text{True Error}}{\text{True Value}}\right| \times 100\,\%$$
$$= \left|\frac{0.77244}{0.97377}\right| \times 100\,\%$$
$$= 79.325\,\%$$

Table 1 Values of Simpson's 1/3 Rule for Example 2 with multiple segments.

n	Approximate Value	E_t	$\lvert\in_t\rvert$ %
2	0.17907	0.79470	81.611
4	0.20133	0.77244	79.325
6	1.0090	−0.035226	3.6175
8	1.2042	−0.23042	23.663
10	1.0954	−0.12167	12.495