Chapter 03.04
Newton-Raphson Method of Solving a Nonlinear Equation-More Examples
Mechanical Engineering

Example 1

A trunnion has to be cooled before it is shrink fitted into a steel hub.

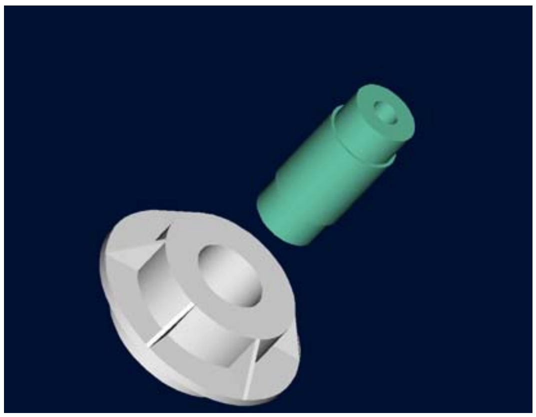

Figure 1 Trunnion to be slid through the hub after contracting.

The equation that gives the temperature T_f to which the trunnion has to be cooled to obtain the desired contraction is given by

$$f(T_f) = -0.50598\times10^{-10}T_f^3 + 0.38292\times10^{-7}T_f^2 + 0.74363\times10^{-4}T_f + 0.88318\times10^{-2} = 0$$

Use the Newton-Raphson method of finding roots of equations to find the temperature T_f to which the trunnion has to be cooled. Conduct three iterations to estimate the root of the above equation. Find the absolute relative approximate error at the end of each iteration and the number of significant digits at least correct at the end of each iteration.

Solution

$$f(T_f) = -0.50598\times10^{-10}T_f^3 + 0.38292\times10^{-7}T_f^2 + 0.74363\times10^{-4}T_f + 0.88318\times10^{-2}$$

$$f'(T_f) = -1.51794\times10^{-10}T_f^2 + 0.76584\times10^{-7}T_f + 0.74363\times10^{-4}$$

Let us assume the initial guess of the root of $f(T_f)=0$ as $T_{f,0} = -100$.

<u>Iteration 1</u>

The estimate of the root is

$$T_{f,1} = T_{f,0} - \frac{f(T_{f,0})}{f'(T_{f,0})}$$

$$= -100 - \frac{\begin{pmatrix} -0.50598\times10^{-10}(-100)^3 + 0.38292\times10^{-7}(-100)^2 \\ +0.74363\times10^{-4}(-100) + 0.88318\times10^{-2} \end{pmatrix}}{1.5179\times10^{-11}(-100)^2 + 0.76582\times10^{-7}(-100) + 0.74363\times10^{-4}}$$

$$= -100 - \frac{1.8290\times10^{-3}}{6.5187\times10^{-5}}$$

$$= -100 - (28.058)$$

$$= -128.06$$

The absolute relative approximate error $|\in_a|$ at the end of Iteration 1 is

$$|\in_a| = \left|\frac{T_{f,1} - T_{f,0}}{T_{f,1}}\right| \times 100$$

$$= \left|\frac{-128.06 - (-100)}{-128.06}\right| \times 100$$

$$= 21.910\%$$

The number of significant digits at least correct is 0, as you need an absolute relative approximate error of less than 5% for one significant digit to be correct in your result.

<u>Iteration 2</u>

The estimate of the root is

$$T_{f,2} = T_{f,1} - \frac{f(T_{f,1})}{f'(T_{f,1})}$$

$$= -128.06 - \frac{\begin{pmatrix} -0.50598\times10^{-10}(-128.06)^3 + 0.38292\times10^{-7} - (128.06)^2 \\ +0.74363\times10^{-4}(-128.06) + 0.88318\times10^{-2} \end{pmatrix}}{\begin{pmatrix} 1.5179\times10^{-10}(-128.06)^2 + 0.76584\times10^{-7}(-128.06) \\ +0.74363\times10^{-4} \end{pmatrix}}$$

$$= -128.06 - \frac{4.3214\times10^{-5}}{6.2067\times10^{-5}}$$

$$= -128.06 - (0.69625)$$

$$= -128.75$$

The absolute relative approximate error $|\in_a|$ at the end of Iteration 2 is

$$|\in_a| = \left|\frac{T_{f,2} - T_{f,1}}{T_{f,2}}\right| \times 100$$

$$= \left|\frac{-128.75 - (-128.06)}{-128.75}\right| \times 100$$

$$= 0.54076\%$$

The number of significant digits at least correct is 1.

<u>Iteration 3</u>

The estimate of the root is

$$T_{f,3} = T_{f,2} - \frac{f(T_{f,2})}{f'(T_{f,2})}$$

$$= -128.75 - \frac{\begin{pmatrix} -0.50598\times10^{-10}(-128.75)^3 + 0.38292\times10^{-7}(-128.75)^2 \\ +0.74363\times10^{-4}(-128.75) + 0.88318\times10^{-2} \end{pmatrix}}{\begin{pmatrix} 1.5179\times10^{-10}(-128.75)^2 + 0.76582\times10^{-7}(-128.75) \\ +0.74363\times10^{-4} \end{pmatrix}}$$

$$= -128.75 - \frac{2.8002\times10^{-8}}{6.1986\times10^{-5}}$$

$$= -128.75 - (4.5175\times10^{-4})$$

$$= -128.75$$

The absolute relative approximate error $|\in_a|$ at the end of Iteration 3 is

$$|\in_a| = \left|\frac{T_{f,3} - T_{f,2}}{T_{f,3}}\right| \times 100$$

$$= \left|\frac{-128.75 - (-128.75)}{-128.75}\right| \times 100$$

$$= 3.5086\times10^{-4}\%$$

Hence the number of significant digits at least correct is given by the largest value of m for which

$$|\in_a| \le 0.5\times10^{2-m}$$

$$3.5086\times10^{-4} \le 0.5\times10^{2-m}$$

$$7.0173\times10^{-4} \le 10^{2-m}$$

$$\log(7.0173\times10^{-4}) \le 2-m$$

$$m \le 2 - \log(7.0173\times10^{-4}) = 5.1538$$

So

$$m = 5$$

The number of significant digits at least correct in the estimated root –128.75 is 5.

NONLINEAR EQUATIONS	
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