Chapter 07.06
Gauss Quadrature Rule for Integration-More Examples
Mechanical Engineering

Example 1

A trunnion of diameter $12.363''$ has to be cooled from a room temperature of $80°\text{F}$ before it is shrink fit into a steel hub (Figure 1).

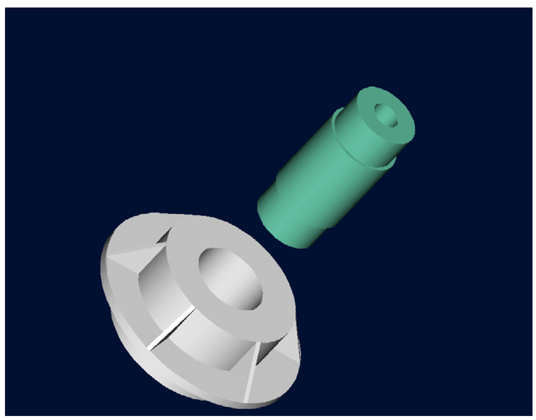

Figure 1 Trunnion to be slid through the hub after contracting.

The equation that gives the diametric contraction, in inches of the trunnion in dry-ice/alcohol (boiling temperature is $-108°\text{F}$) is given by:

$$\Delta D = 12.363 \int_{80}^{-108} \left(-1.2278 \times 10^{-11} T^2 + 6.1946 \times 10^{-9} T + 6.015 \times 10^{-6}\right) dT$$

a) Use two-point Gauss Quadrature Rule to find the contraction.
b) Find the absolute relative true error.

Solution

a) First, change the limits of integration from $[80, -108]$ to $[-1, 1]$ using

$$a = 80$$

$$b = -108$$

$$\int_a^b f(T)dT = \frac{b-a}{2} \int_{-1}^{1} f\left(\frac{b-a}{2}T + \frac{b+a}{2}\right) dT$$

gives

$$\int_{80}^{-108} f(T)dT = \frac{-108-80}{2}\int_{-1}^{1} f\left(\frac{-108-80}{2}T + \frac{-108+80}{2}\right)dT$$

$$= -94\int_{-1}^{1} f(-94T-14)dT$$

Next, get weighting factors and function argument values for the two point rule,

$c_1 = 1.0000$
$T_1 = -0.57735$
$c_2 = 1.0000$
$T_2 = 0.57735$

Now we can use the Gauss Quadrature formula

$$-94\int_{-1}^{1} f(-94T-14)dx \approx -94\left[c_1 f(-94T_1-14) + c_2 f(-94T_2-14)\right]$$
$$\approx -94\left[f(-94(-0.57735)-14) + f(-94(0.57735)-14)\right]$$
$$\approx -94\left[f(40.271) + f(-68.271)\right]$$
$$\approx -94\left[(7.7201\times10^{-5}) + (6.8428\times10^{-5})\right]$$
$$\approx -0.013689 \text{ in}$$

since

$$f(40.271) = 12.363\left(-1.2278\times10^{-11}(40.271)^2 + 6.1946\times10^{-9}(40.271) + 6.015\times10^{-6}\right)$$
$$= 7.7201\times10^{-5}$$
$$f(-68.271) = 12.363\left(-1.2278\times10^{-11}(-68.271)^2 + 6.1946\times10^{-9}(-68.271) + 6.015\times10^{-6}\right)$$
$$= 6.8428\times10^{-5}$$

b) The absolute relative true error, $|\in_t|$, is (Exact value = -0.013689)

$$|\in_t| = \left|\frac{-0.013689-(-0.013689)}{-0.013689}\right|\times100\,\%$$
$$= 4.0309\times10^{-10}$$

Example 2

A trunnion of diameter $12.363''$ has to be cooled from a room temperature of $80°\text{F}$ before it is shrink fit into a steel hub (Figure 1). The equation that gives the diametric contraction, in inches of the trunnion in dry-ice/alcohol (boiling temperature is $-108°\text{F}$) is given by:

$$\Delta D = 12.363\int_{80}^{-108}\left(-1.2278\times10^{-11}T^2 + 6.1946\times10^{-9}T + 6.015\times10^{-6}\right)dT$$

a) Use three-point Gauss Quadrature Rule to find the contraction.
b) Also, find the absolute relative true error.

Solution

a) First, change the limits of integration from $[80, -108]$ to $[-1, 1]$ using

$$a = 80$$
$$b = -108$$
$$\int_a^b f(T)dT = \frac{b-a}{2}\int_{-1}^{1} f\left(\frac{b-a}{2}T + \frac{b+a}{2}\right)dT$$

gives

$$\int_{80}^{-108} f(T)dT = \frac{-108-80}{2}\int_{-1}^{1} f\left(\frac{-108-80}{2}T + \frac{-108+80}{2}\right)dT$$
$$= -94\int_{-1}^{1} f(-94T-14)dT$$

The weighting factors and function argument values are

$$c_1 = 0.55556$$
$$T_1 = -0.77460$$
$$c_2 = 0.88889$$
$$T_2 = 0.0000$$
$$c_3 = 0.55556$$
$$T_3 = 0.77460$$

and the formula is

$$-94\int_{-1}^{1} f(-94T-14)dT \approx -94\left[c_1 f(-94T_1 - 14) + c_2 f(-94T_2 - 14) + c_3 f(-94T_3 - 14)\right]$$
$$\approx -94\begin{bmatrix} 0.55556\times f(-94(-0.77460)-14) \\ +0.88889\times f(-94(0.0000)-14) \\ +0.55556\times f(-94(0.77460)-14) \end{bmatrix}$$
$$\approx -94\begin{bmatrix} 0.55556 f(58.812) + 0.88889 f(-14.000)) \\ +0.55556 f(-86.812) \end{bmatrix}$$
$$\approx -94\begin{bmatrix} 0.55556(7.8342\times 10^{-5}) + 0.88889(7.3262\times 10^{-5}) \\ +0.55556(6.6571\times 10^{-5}) \end{bmatrix}$$
$$\approx -0.013689 \text{ in}$$

since

$$f(58.812) = 12.363\left(-1.2278\times 10^{-11}(58.812)^2 + 6.1946\times 10^{-9}(58.812) + 6.015\times 10^{-6}\right)$$
$$= 7.8343\times 10^{-5}$$

$$f(-14.000) = 12.363\left(-1.2278\times 10^{-11}(-14.000)^2 + 6.1946\times 10^{-9}(-14.000) + 6.015\times 10^{-6}\right)$$
$$= 7.3262\times 10^{-5}$$

$$f(-86.812) = 12.363\left(-1.2278\times 10^{-11}(-86.812)^2 + 6.1946\times 10^{-9}(-86.812) + 6.015\times 10^{-6}\right)$$
$$= 6.6571\times 10^{-5}$$

b) The absolute relative true error, $\left|\in_t\right|$, is (Exact value = -0.013689)

$$\left|\in_t\right| = \left|\frac{\text{True Value} - \text{Approximate Value}}{\text{True Value}}\right| \times 100\,\%$$

$$\left|\in_t\right| = \left|\frac{-0.013689 - (-0.013689)}{-0.013689}\right| \times 100\,\%$$

$$= 0.0000\,\%$$