True Error

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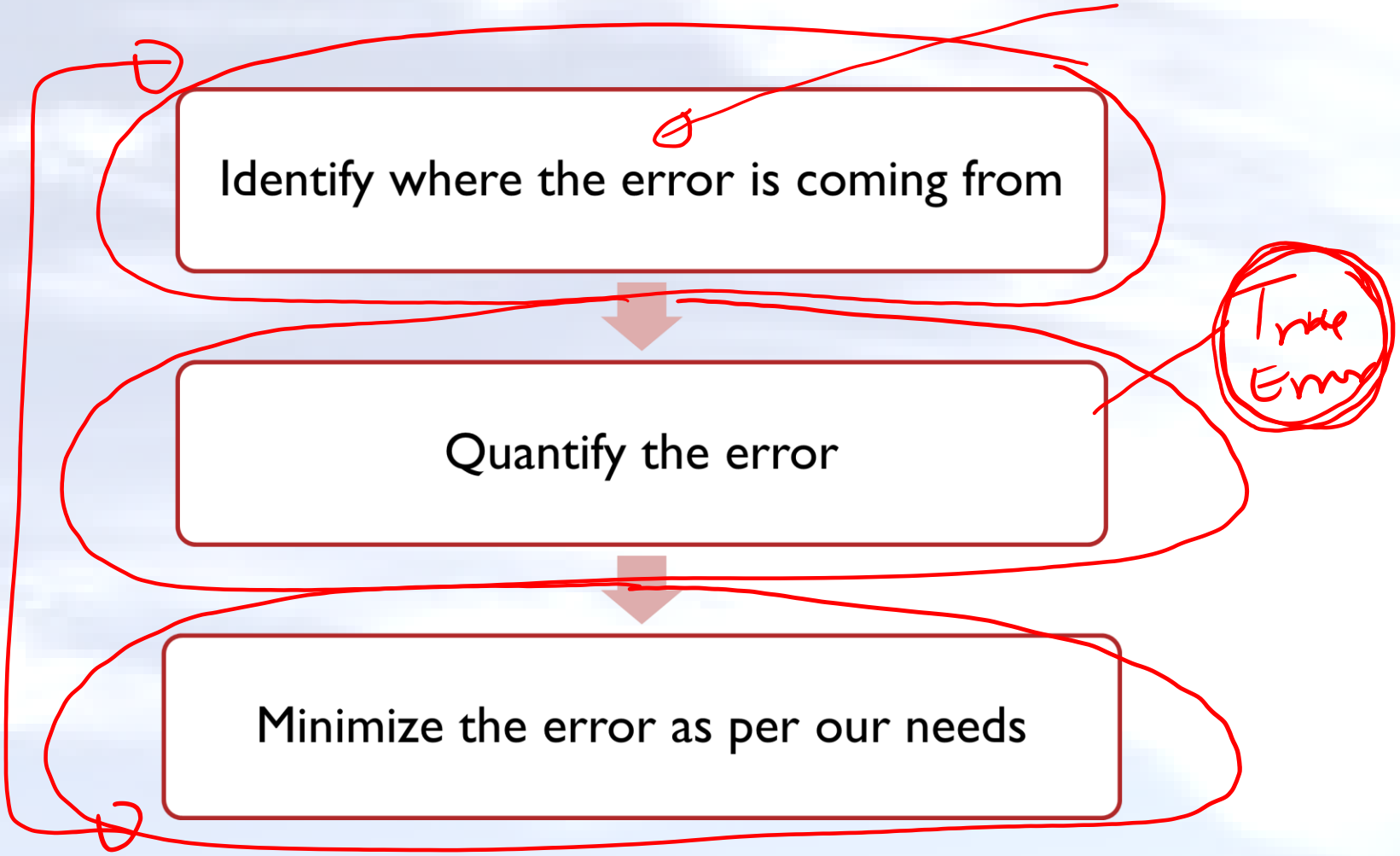
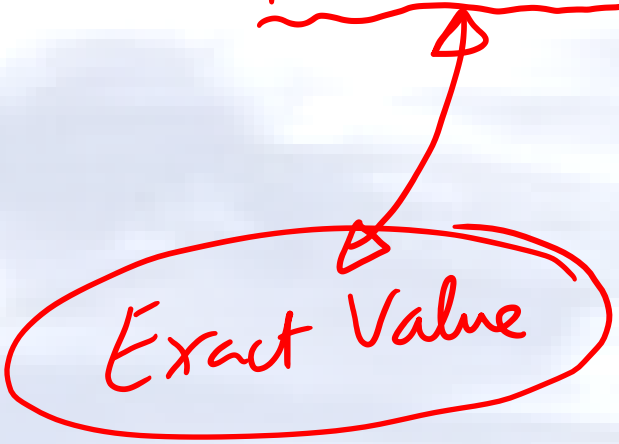
- Go to http://nm.MathForCollege.com
- Click on Quantifying Errors

How to deal with errors?



True Error (E_t) = True Value − Approx. Value



Example of true error

The derivative of a function $f(x)$ at a particular value of x can be approximately calculated by

$$f'(x) \approx \frac{f(x+h) - f(x)}{h}$$

For $f(x) = 7e^{0.5x}$ and $h = 0.3$, find

a) the approximate value of $f'(2)$

b) the true value of $f'(2)$

c) the true error for part (a)

$$f'(x) \simeq \frac{f(x+h)-f(x)}{h}$$

$x = 2, \quad h = 0.3$

$$f'(2) \simeq \frac{f(2+0.3)-f(2)}{0.3}$$

$$= \frac{f(2.3)-f(2)}{0.3}$$

$$= \frac{7e^{0.5(2.3)} - 7e^{0.5(2)}}{0.3}$$

$$= 10.263$$

b) $f''(2) = ?$ (Exact)

$f(x) = 7e^{0.5x}$

$\frac{d}{dx}(ce^{ax}) = cae^{ax}$

$\frac{d}{dx}(7e^{0.5x}) = 7(0.5)e^{0.5x}$

$= 3.5e^{0.5x}$

$f'(x) = 3.5e^{0.5x}$

$f'(2) = 3.5e^{0.5(2)}$

$= \underline{9.5140}$

c) E_t = True value − Approx. value

$= 9.5140 - 10.263$

$= -0.749$

If $f(x) = 7\ e^{0.5x}$

Instead we have $f(x) = 7 * 10^{-6} e^{0.5x}$

$E_t = -0.749 * 10^{-6}$ ✓

$$\text{Relative True Error } (\epsilon_t) = \frac{\text{True Error}}{\text{True value}}$$

Example of relative true error

The derivative of a function $f(x)$ at a particular value of x can be approximately calculated by

$$f'(x) \approx \frac{f(x+h) - f(x)}{h}$$

For $f(x) = 7e^{0.5x}$ and $h = 0.3$, find the relative true error in finding $f'(2)$.

$E_t = -0.749$

True value $= 9.5140$

$$\epsilon_t = \frac{E_t}{\text{True val.}} = -\frac{0.749}{9.5140}$$

$$= -0.078726$$

$$\epsilon_t = -0.078726$$

$$\epsilon_t = -0.078726 * 100\%$$

$$= 7.8726\%$$

Absolute Relative True Error

$$|\epsilon_t| = |-0.078726|$$

$$= 0.078726$$

or

$$7.8726\%$$

$f(x) = 7e^{0.5x}$ $\quad$ $f(x) = 7 * 10^{-6} e^{0.5x}$

END

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