

Floating-Point Representation Background



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Transforming Numerical Methods Education for STEM Undergraduates



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- Go to <http://nm.MathForCollege.com>
- Click on Floating-Point Binary Representation of Numbers



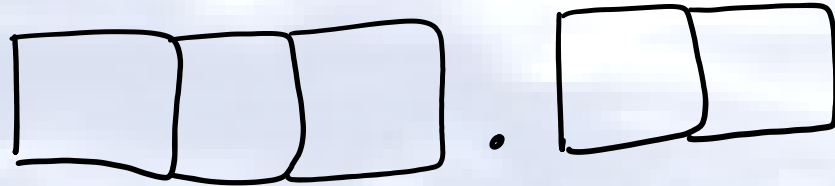
Base-10

256.78 (decimal, fixed-point)

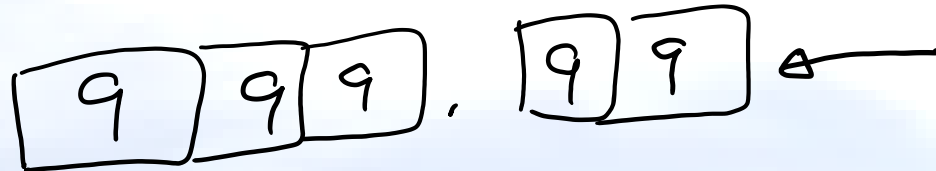
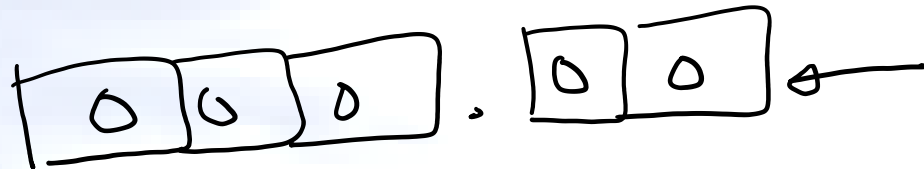
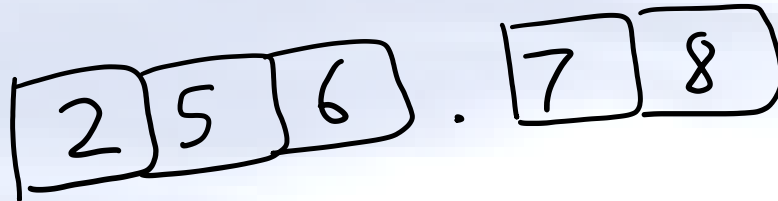
2.5678×10^2 (scientific, floating-point format)



Fixed-point format



256.78



Errors in fixed-point format

256.786^{↓ ↓}
↑

□ □ □ . □ □

1256.78 (Chopping)

$$E_t = 256.786 - 256.78 = 0.006 \checkmark$$

$$|E_t| = \left| \frac{0.006}{256.786} \right| = 0.000023366 \checkmark$$

3.546

□ □ □ . □ □

$$E_t = 0.006 \checkmark$$

$$|E_t| = 0.0016920 \checkmark$$

0.016

□ □ □ . □ □

$$E_t = 0.006 \checkmark$$

$$|E_t| = 0.375 \checkmark$$

$$\|E_t\| < 0.01 = 10^{-2}$$

$$|E_t| \leq 10^{-p}$$



Errors in floating-pt. format

□□□.□□ fixed-pt.

□.□□□ □
mantissa exponent

1.000

0

1.0

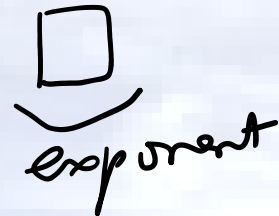
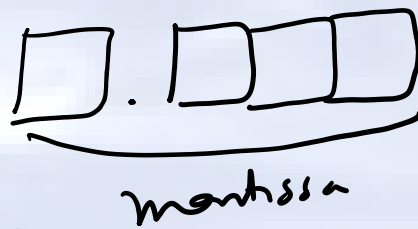
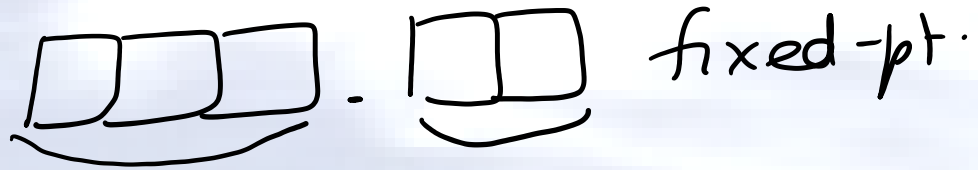
9.999

9

9.999 × 10⁹



Floating Point Format



$1.000 \times 10^0 = 1$



9.999×10^9



Errors in floating point format
256.78 2.5678×10^2

$\boxed{2.567} \quad \boxed{2}$

$$E_t = 256.78 - 2.567 \times 10^2 = 0.08 \checkmark$$

$$|E_t| = \left| \frac{0.08}{256.78} \right| = 0.00031155 \checkmark$$

576329.78

$$E_t = 29.78 \checkmark$$

$$|E_t| = 0.000051672 \checkmark$$

576399.99

$$E_t = 99.99 \checkmark$$

$$|E_t| = 0.00017347 \checkmark$$

1.0009999

$$E_t = 0.0009999 \checkmark$$

$$|E_t| = 0.00099890 \checkmark$$

$$|E_t| < 0.001 \rightarrow 10^{-3} = 10^{-4}$$

$$|E_t| < 10^{1-p}$$



END



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Numerical Methods for STEM undergraduate

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Committed to bringing numerical methods to the
undergraduate

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