Numerical Differentiation of Continuous Functions - First Derivative Backward Divided Difference

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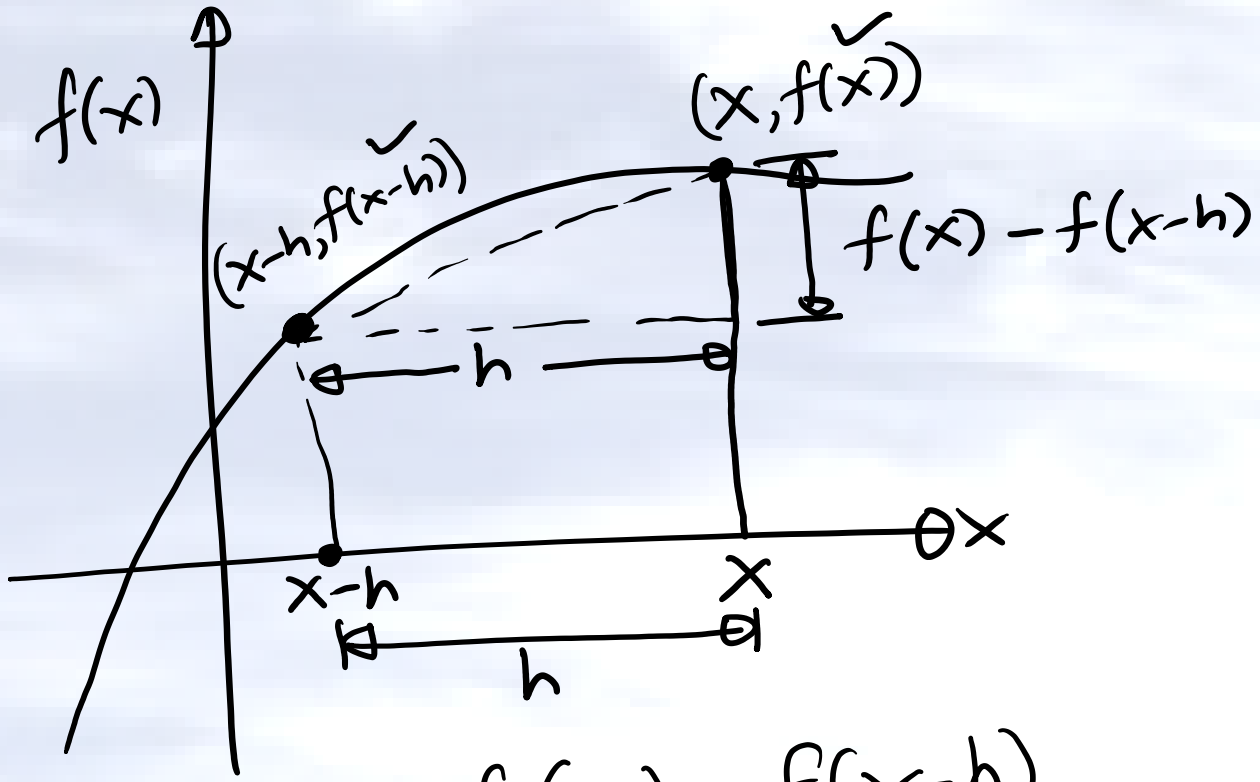
BDD Method $f'(x)$



$$f'(x) \simeq \frac{f(x) - f(x-h)}{h}$$

Example of backward divided difference to find $f'(x)$

For $f(x) = 7x^4$ and a step size of $h = 0.16$, use the backward divided difference formula to find

a) the approximate value of $f'(3)$
b) the true value of $f'(3)$
c) the true error for part (a)
d) discuss trends in true error as a function of step size
e) discuss absolute relative approximate error relation to significant digits correct as step size is decreased

a) $$f'(x) \simeq \frac{f(x)-f(x-h)}{h}$$

$x = 3,\ h = 0.16,\ f(x) = 7x^4$

$$f'(3) \simeq \frac{f(3)-f(3-0.16)}{0.16} = \frac{f(3)-f(2.84)}{0.16}$$

$$= \frac{7(3)^4 - 7(2.84)^4}{0.16}$$

$$= 697.6417$$

b) $f(x) = 7x^4$

$f'(x) = 7(4x^3) = 28x^3$

$f'(3) = 28(3)^3 = 756$

c) $E_t = \text{True Value} - \text{Approx. Value}$

$= 756 - 697.6417$

$= 58.3583$

d) Exact Value = 756

h	f'(3)	E_t
0.16	697.6417	58.3583
0.08	726.2940	29.7060
0.04	741.0140	14.9860

e)

h	$f'(3)$	E_a	$\lvert E_a \rvert$
0.16	697.6417	—	—
0.08	726.2940	28.6523	3.9450%
0.04	741.0140	14.7199	1.9865%

$\leq 5\%$ but not $\leq 0.5\%$

$f'(3) \simeq 741.0140$

END

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