Numerical Differentiation of Continuous Functions - First Derivative Central Divided Difference

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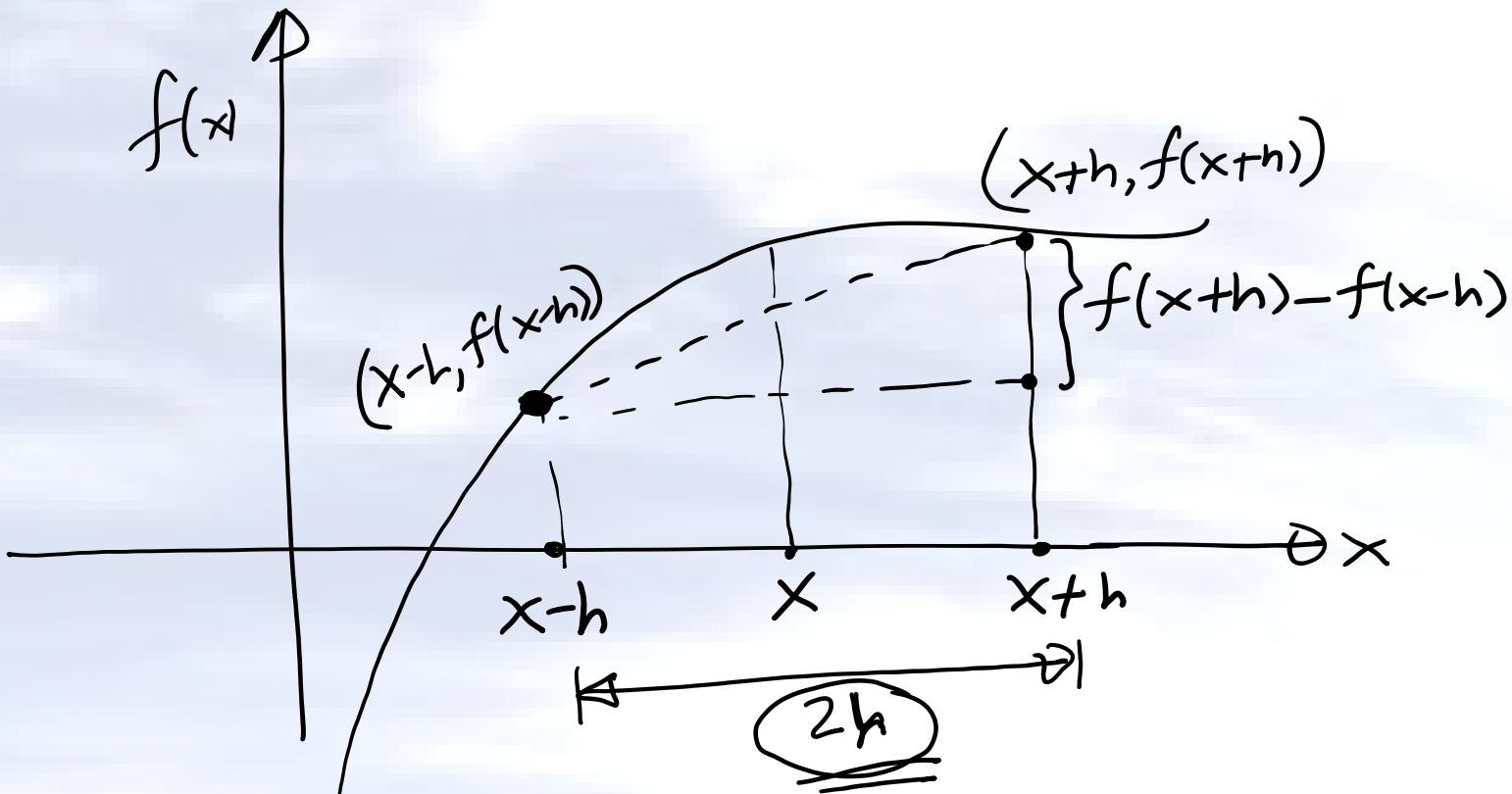

$$f'(x) \simeq \frac{f(x+h)-f(x-h)}{2h}$$

CDD

Example of central divided difference to find $f'(x)$

For $f(x) = 7x^4$ and a step size of $h = 0.16$, use the central divided difference formula to find

a) the approximate value of $f'(3)$
b) the true value of $f'(3)$
c) the true error for part (a)
d) discuss trends in true error as a function of step size
e) discuss absolute relative approximate error relation to significant digits correct as step size is decreased

a) $$f'(x) \simeq \frac{f(x+h) - f(x-h)}{2h}$$

$$x = 3, \; h = 0.16$$

$$f'(3) = \frac{f(3+0.16) - f(3-0.16)}{2(0.16)}$$

$$= \frac{f(3.16) - f(2.84)}{0.32}$$

$$f(x) = 7x^4$$

$$f'(3) = \frac{7(3.16)^4 - 7(2.84)^4}{0.32}$$

$$= 758.1504$$

b) True Value: $f'(3) = ?$

$$f(x) = 7x^4$$

$$f'(x) = 7(4x^3) = 28x^3$$

$$f'(3) = 28(3)^3 = \underline{\underline{756}}$$

c) E_t = True Value − Approx. Value

$$= 756 - 758.1504$$

$$= -2.1504$$

d)

h	$f'(3)$	E_t
0.16	758.1504	-2.1504
0.08	756.5376	-0.5376
0.04	756.1344	-0.1344

e)

h	$f'(3)$	E_a	$\lvert\epsilon_a\rvert$
0.16	758.1504	—	—
0.08	756.5376	−1.628	0.2132%
0.04	756.1344	−0.4032	0.0533%

$\leq 5\%$ — trust 1 sign dgt

$\leq 0.5\%$ — " 2 " "

$\leq 0.05\%$ — no.

$f'(3) = 756.1344$

END

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