Numerical Differentiation of Continuous Functions - First Derivative Forward Divided Difference

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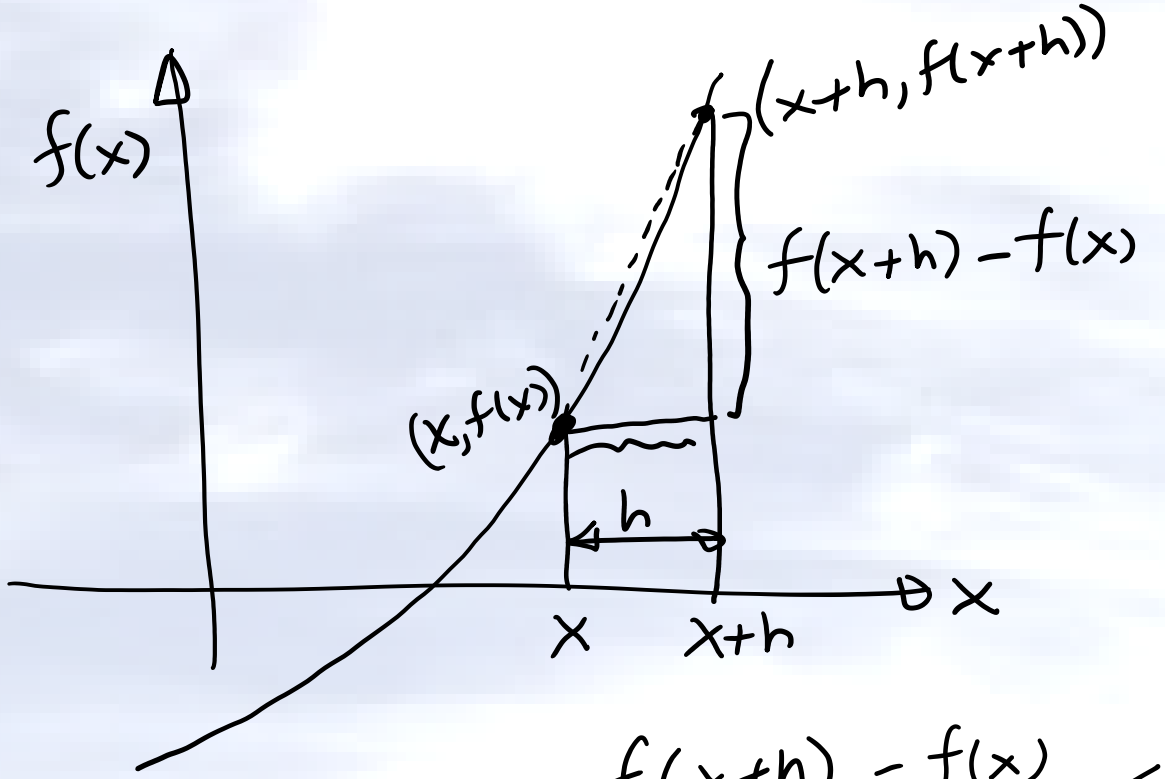

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(x) \simeq \frac{f(x+h) - f(x)}{h}$$

Example of forward divided difference to find $f'(x)$

For $f(x) = 7x^4$ and a step size of $h = 0.16$, use the forward divided difference formula to find

a) the approximate value of $f'(3)$
b) the true value of $f'(3)$
c) the true error for part (a)
d) discuss trends in true error as a function of step size
e) discuss absolute relative approximate error relation to significant digits correct as step size is decreased

a) $f'(x) \simeq \dfrac{f(x+h) - f(x)}{h}$

$x = 3,\ h = 0.16,\ f(x) = 7x^4$

$f'(3) \simeq \dfrac{f(3+0.16) - f(3)}{0.16}$

$= \dfrac{f(3.16) - f(3)}{0.16}$

$= \dfrac{7(3.16)^4 - 7(3)^4}{0.16}$

$= 818.6591$

b) $f(x) = 7x^4$

$f'(x) = 7(4x^3)$

$= 28x^3$

$f'(3) = 28(3)^3 = 756$

c) $E_t =$ True Value - Appr. Value

$= 756 - 818.6591$

$= -62.6591$

d)

h	$f''(3)$	E_t
0.16	818.6591	-62.6591
0.08	786.7812	−30.7812
0.04	771.2548	−15.2548

e)

h	$f'(3)$	E_a	$\lvert E_a \rvert$
0.16	818.6591	—	—
0.08	786.7812	−31.8779	4.0517%
0.04	771.2548	−15.5263	2.0131%

2.0131% ≤ 5%

but not ≤ 0.5%

$h = 0.04 \rightarrow$ 771.2548

END

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