

# Numerical Differentiation of Continuous Functions Second Derivative: Theory



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# Taylor Series

$$f(x+h) = f(x) + f'(x)(h) + \frac{f''(x)}{2!}(h)^2 + \frac{f'''(x)}{3!}(h)^3 + \frac{f''''(x)}{4!}(h)^4 + \dots \quad (1)$$

$$f(x-h) = f(x) + f'(-h) + \frac{f''(-h)^2}{2!} + \frac{f'''(-h)^3}{3!} + \frac{f''''(-h)^4}{4!} + \dots$$

$$f(x-h) = f(x) - f'(x)h + \frac{f''(x)}{2!}h^2 - \frac{f'''(x)}{3!}h^3 + \frac{f''''(x)}{4!}h^4 + \dots \quad (2)$$

# Derivation of $f''(x)$

$$\begin{aligned}
 f(x+h) &= f(x) + f'(x)h + \frac{f''(x)}{2!}h^2 + \frac{f'''(x)}{3!}h^3 + \frac{f''''(x)}{4!}h^4 + \dots \quad \text{--- ①} \\
 f(x-h) &= f(x) - f'(x)h + \frac{f''(x)}{2!}h^2 - \frac{f'''(x)}{3!}h^3 + \frac{f''''(x)}{4!}h^4 + \dots \quad \text{--- ②}
 \end{aligned}$$

$$f(x+h) + f(x-h) = 2f(x) + 2\frac{f''(x)}{2!}h^2 + 2\frac{f''''(x)}{4!}h^4 + \dots$$

$$\frac{f(x+h) + f(x-h) - 2f(x)}{h^2} = \frac{2\frac{f''''(x)}{4!}h^4}{h^2} - \dots = f''(x)h^2 - \dots$$

$$f''(x) = \frac{f(x+h) - 2f(x) + f(x-h)}{h^2} - 2\frac{f''''(x)}{4!}h^2 - \dots$$

$$f''(x) \approx \frac{f(x+h) - 2f(x) + f(x-h)}{h^2}$$

END



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