LU Decomposition Method: Example

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Example

Use the LU decomposition method to solve the following simultaneous linear equations.

$$\begin{bmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 106.8 \\ 177.2 \\ 279.2 \end{bmatrix}$$

The LU Decomposition of the coefficient matrix is given

$$\begin{bmatrix} 25 & 5 & 1 \\ 64 & 8 & 1 \\ 144 & 12 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 2.56 & 1 & 0 \\ 5.76 & 3.5 & 1 \end{bmatrix} \begin{bmatrix} 25 & 5 & 1 \\ 0 & -4.8 & -1.56 \\ 0 & 0 & 0.7 \end{bmatrix}$$

$$[A][X] = [C]$$

$$[A] = [L][U]$$

$$[L][Z] = [C] \quad \text{— ①}$$

$$[U][X] = [Z] \quad \text{— ②}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 2.56 & 1 & 0 \\ 5.76 & 3.5 & 1 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} 106.8 \\ 177.2 \\ 279.2 \end{bmatrix}$$

$[L][Z]=[C]$

Forward substitution.

$z_1 = 106.8$

$2.56 z_1 + z_2 = 177.2$

$5.76 z_1 + 3.5 z_2 + z_3 = 279.2$

$$z_2 = \frac{177.2 - 2.56 z_1}{1} = 177.2 - 2.56(106.8)$$

$$= -96.208$$

$$z_3 = \frac{279.2 - 3.5 z_2 - 5.76 z_1}{1}$$

$$= 279.2 - 3.5(-96.208) - 5.76(177.2)$$

$$= 0.76$$

$$(z) = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} = \begin{bmatrix} 106.8 \\ -96.208 \\ 0.76 \end{bmatrix}$$

$[U][X] = [z]$ — Back substitution.

$$\begin{bmatrix} 25 & 5 & 1 \\ 0 & -4.8 & -1.56 \\ 0 & 0 & 0.7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 106.8 \\ -96.208 \\ 0.76 \end{bmatrix}$$

$$0.7x_3 = 0.76 \Rightarrow x_3 = \frac{0.76}{0.7} = 1.0857$$

$$-4.8x_2 - 1.56x_3 = -96.208$$

$$x_2 = \frac{-96.208 + 1.56x_3}{-4.8} = \frac{-96.208 + 1.56(1.0857)}{-4.8} = 19.691$$

$$25x_1 + 5x_2 + x_3 = 106.8$$

$$x_1 = \frac{106.8 - 5x_2 - x_3}{25} = \frac{106.8 - 5(19.691) - (1.0857)}{25} = 0.29048$$

$$(X) = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0.29048 \\ 19.691 \\ 1.0857 \end{bmatrix}$$

END

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