Direct (Vandermonde) Method of Interpolation
Quadratic Interpolation

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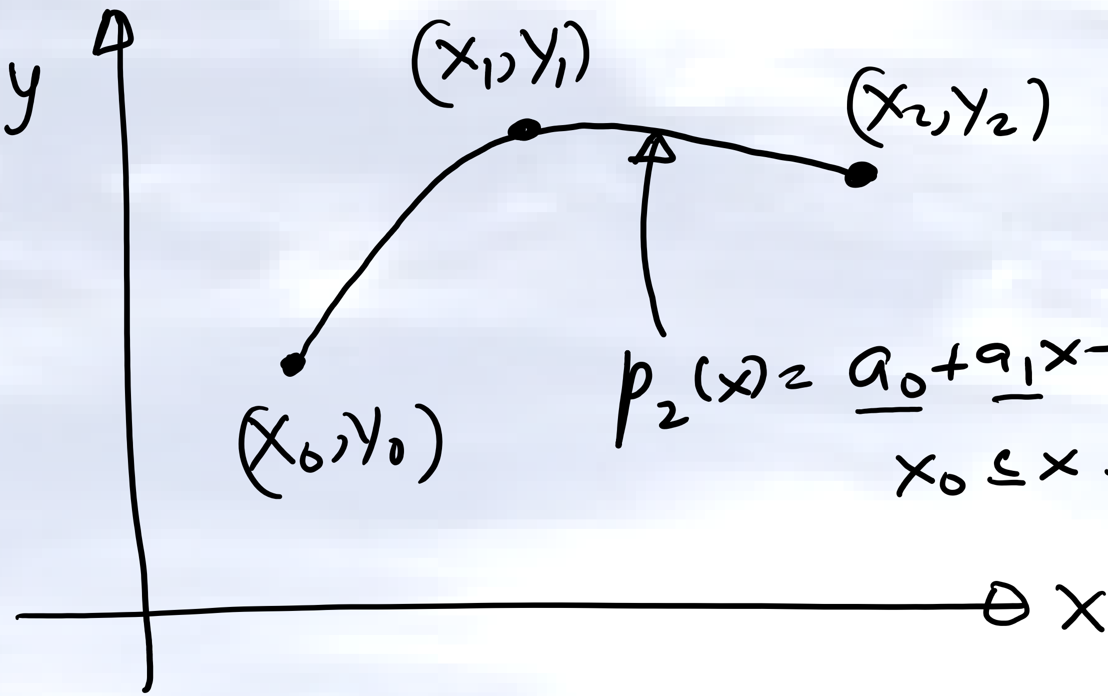
Quadratic Interpolation



$$p_2(x) = a_0 + a_1 x + a_2 x^2, \quad x_0 \le x \le x_2$$

Example of direct (Vandermonde) method of quadratic interpolation

The upward velocity of a rocket is given as a function of time.

t (s)	$v(t)$ (m/s)
0	0
10	227.04
15	362.78
20	517.35
22.5	602.97
30	901.67

$|16-10| = 6$

$|22.5-16| = 6.5$

a) Estimate the velocity at t=16 seconds using the direct method of interpolation with a second-order polynomial.

b) Find the absolute relative approximate error for the second-order polynomial approximation of the velocity at t=16 seconds.

c) Using the second-order polynomial interpolant for velocity from part (a), find the distance covered by the rocket from t=11 s to t=16 s.

d) Using the second-order polynomial interpolant for velocity from part (a), find the acceleration of the rocket at t=16 s.

a) $v(t) = a_0 + a_1 t + a_2 t^2, \quad 10 \le t \le 20$

$$v(10) = a_0 + a_1(10) + a_2(10)^2 = 227.04 \quad \text{— ①}$$

$$v(15) = a_0 + a_1(15) + a_2(15)^2 = 362.78 \quad \text{— ②}$$

$$v(20) = a_0 + a_1(20) + a_2(20)^2 = 517.35 \quad \text{— ③}$$

$$1a_0 + 10a_1 + 100a_2 = 227.04$$

$$1a_0 + 15a_1 + 225a_2 = 362.78$$

$$1a_0 + 20a_1 + 400a_2 = 517.35$$

$$\begin{bmatrix} 1 & 10 & 100 \\ 1 & 15 & 225 \\ 1 & 20 & 400 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \end{bmatrix} = \begin{bmatrix} 227.04 \\ 362.78 \\ 517.35 \end{bmatrix}$$

$$a_0 = 12.050,\ a_1 = 17.733,\ a_2 = 0.37760$$

$$v(t) = a_0 + a_1 t + a_2 t^2$$

$$= 12.050 + 17.733t + 0.37760t^2, \quad 10 \le t \le 20$$

$$v(16) = 12.050 + 17.733(16) + 0.37760(16)^2$$

$$= 392.19 \text{ m/s}$$

b) $|\epsilon_a| = ?$

$v(16) \simeq 392.19$ (2nd order poly)

$v(16) \simeq 393.70$ (1st order poly)

$$|\epsilon_a| = \left|\frac{392.19 - 393.70}{392.19}\right| \times 100$$

$$= \underline{0.38410\%}$$

c) 11s to 16s - distance covered

$$v(t) = 12.050 + 17.733t + 0.37660t^2, \quad 10 \le t \le 20$$

$$s(16) - s(11) = \int_{11}^{16} v(t)\,dt$$

$$= \int_{11}^{16} (12.050 + 17.733t + 0.37660t^2)\,dt$$

$$= \left[12.050t + \frac{17.733t^2}{2} + \frac{0.37660t^3}{3}\right]_{11}^{16}$$

$$= 1604.3 \text{ m}$$

d) $a(16) = ?$

$$a(t) = \frac{dv(t)}{dt}$$

$$v(t) = 12.050 + 17.733t + 0.37660t^2, \quad 10 \le t \le 20$$

$$a(t) = \frac{d}{dt}(v(t)) = \frac{d}{dt}(12.050 + 17.733t + 0.37660t^2)$$

$$= 17.733 + 2(0.37660)t, \quad 10 \le t \le 20$$

$$a(16) = 17.733 + 2(0.37660)(16)$$

$$= 29.784 \text{ m/s}^2$$

END

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