Deriving the General Straight-Line Regression Model

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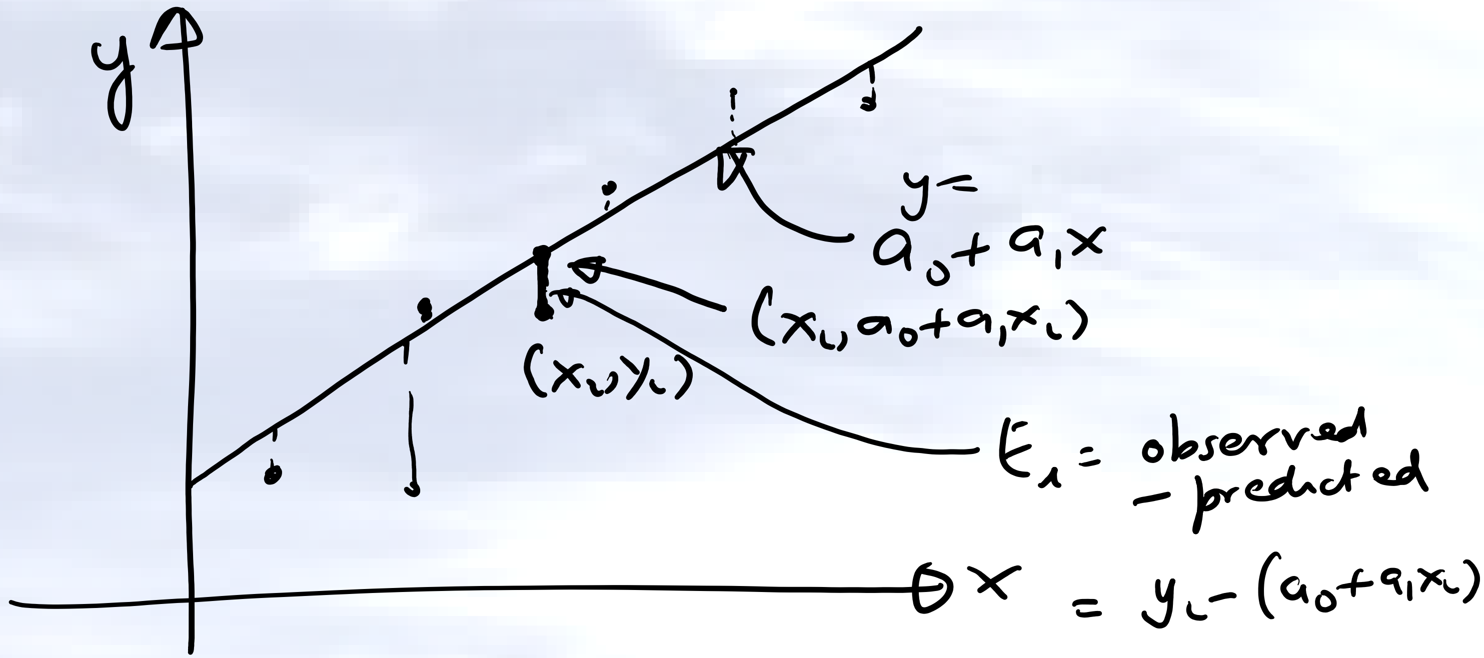
Given $(x_1, y_1), \ldots, (x_n, y_n)$, best fit $y = a_0 + a_1 x$ to the data.



$$S_r = \sum_{i=1}^{n} E_i^2 = \sum_{i=1}^{n} \left(y_i - (a_0 + a_1 x_i)\right)^2$$

$$= \boxed{\sum_{i=1}^{n} (y_i - a_0 - a_1 x_i)^2}$$

$$S_r = \sum_{i=1}^{n} (y_i - a_0 - a_1 x_i)^2$$

make it as small as possible

First derivative part

$$\frac{\partial S_r}{\partial a_0} = \sum_{i=1}^{n} 2(y_i - a_0 - a_1 x_i)(-1) = 0$$

$$\frac{\partial S_r}{\partial a_1} = \sum_{i=1}^{n} 2(y_i - a_0 - a_1 x_i)(-x_i) = 0$$

$$0 \quad -2 \sum_{i=1}^{n} y_i + 2 \sum_{i=1}^{n} a_0 + 2 \sum_{i=1}^{n} a_1 x_i = 0$$

$$0 \quad -2 \sum_{i=1}^{n} x_i y_i + 2 \sum_{i=1}^{n} a_0 x_i + 2 \sum_{i=1}^{n} a_1 x_i^2 = 0$$

Divide both eqns by 2

$$-\sum_{i=1}^{n} y_i + \sum_{i=1}^{n} a_0 + \sum_{i=1}^{n} a_1 x_i = 0$$

$$-\sum_{i=1}^{n} x_i y_i + \sum_{i=1}^{n} a_0 x_i + \sum_{i=1}^{n} a_1 x_i^2 = 0$$

or

$$n a_0 + a_1 \sum_{i=1}^{n} x_i = \sum_{i=1}^{n} y_i \quad \text{--- (1)}$$

$$a_0 \sum_{i=1}^{n} x_i + a_1 \sum_{i=1}^{n} x_i^2 = \sum_{i=1}^{n} x_i y_i \quad \text{--- (2)}$$

Multiply eqn (1) by $\sum_{i=1}^{n} x_i$, and eqn (2) by n

and subtract,

$$a_1 = \frac{n \sum_{i=1}^{n} x_i y_i - \sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i}{n \sum_{i=1}^{n} x_i^2 - \left(\sum x_i\right)^2}$$

Eqn (1)

$$n a_0 + a_1 \sum_{i=1}^{n} x_i = \sum_{i=1}^{n} y_i$$

$$n a_0 = \sum_{i=1}^{n} y_i - a_1 \sum_{i=1}^{n} x_i$$

$$a_0 = \frac{\sum_{i=1}^{n} y_i}{n} - a_1 \frac{\sum_{i=1}^{n} x_i}{n}$$

$$\boxed{a_0 = \bar{y} - a_1 \bar{x}}$$

$$\boxed{a_1 = \frac{n \sum_{i=1}^{n} x_i y_i - \sum_{i=1}^{n} x_i \sum_{i=1}^{n} y_i}{n \sum_{i=1}^{n} x_i^2 - \left(\sum_{i=1}^{n} x_i\right)^2}}$$

$$\boxed{y = a_0 + a_1 x}$$

Second derivative test tell us that a_0, a_1 values found correspond to a local minimum.

✓ S_r is a continuous function of a_0 and a_1; $\underline{\frac{\partial S_r}{\partial a_0} = 0; \frac{\partial S_r}{\partial a_1} = 0}$ only yields one solution.

That implies that a_0, a_1 values found correspond to an absolute minimum.

$\boxed{\text{END}}$

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