Adequacy of Linear Regression Models Part: Check Three: Coefficient of Determination

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Transforming Numerical Methods Education for STEM Undergraduates

UNIVERSITY OF SOUTH FLORIDA

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- Click on Adequacy of Regression Models

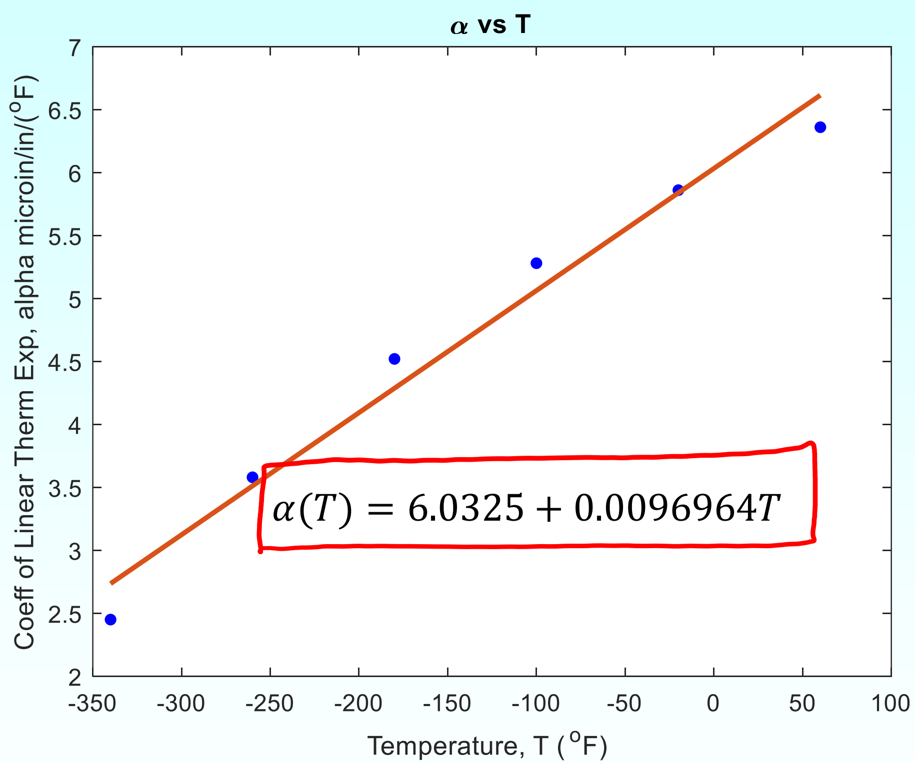
Four checks

1. Does the model look like it explains the data?
2. Do 95%of the residuals fall within ±2 of standard error of estimate?
3. Is the coefficient of determination acceptable?
4. Does the model meet the assumption of random errors?

3. Is the coefficient of determination acceptable?

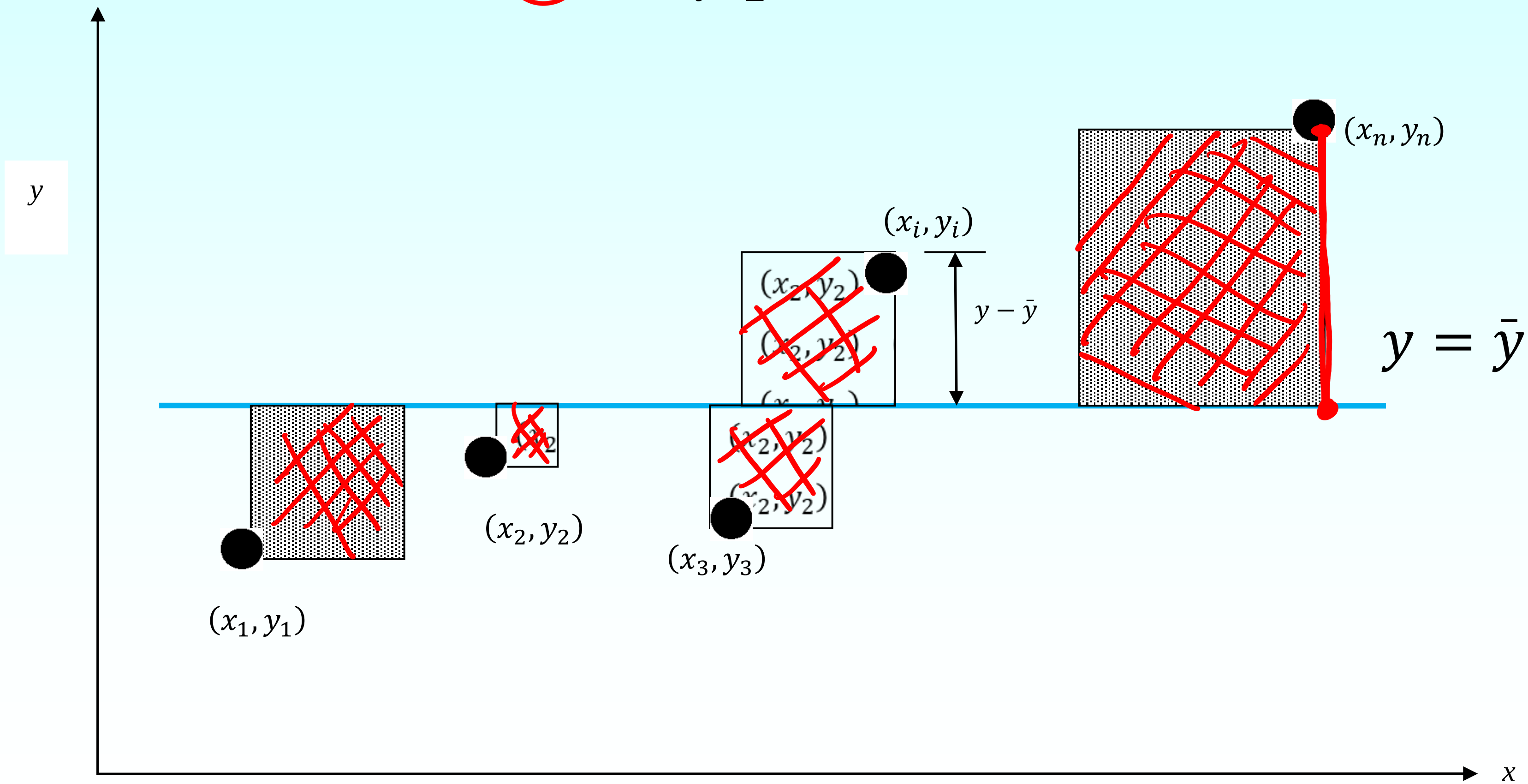
Data and Model

T_i (°F)	α_i μm/m/°F
-340	2.45
-260	3.58
-180	4.52
-100	5.28
-20	5.86
60	6.36



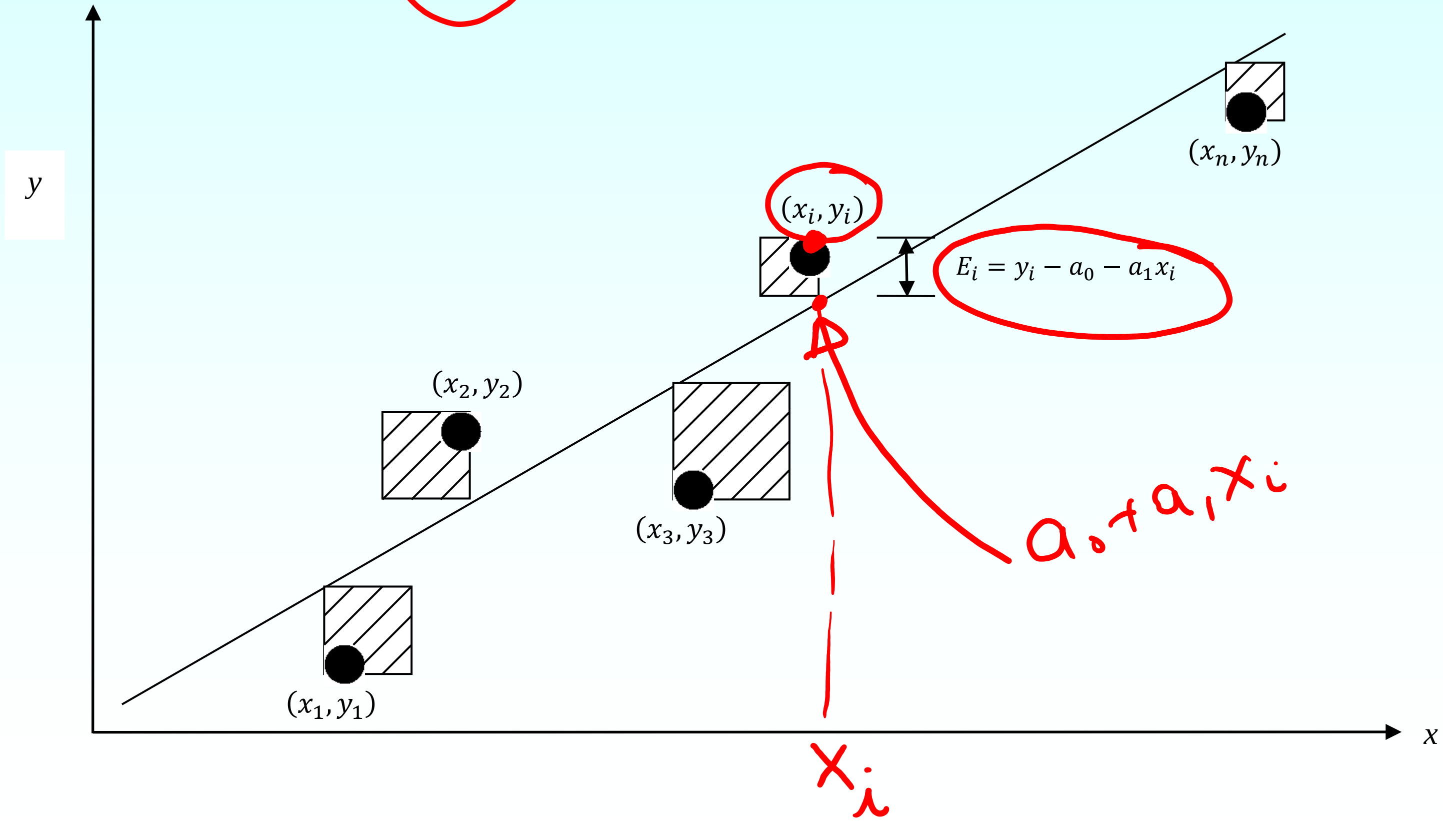
Sum of square of residuals between data and mean

$$S_t = \sum_{i=1}^{n} (\alpha_i - \bar{\alpha})^2$$



Sum of square of residuals between observed and predicted

$$S_r = \sum_{i=1}^{n} (\alpha_i - a_0 - a_1 T_i)^2$$



Coefficient of determination

$$S_t = \sum_{i=1}^{n} (\alpha_i - \bar{\alpha})^2$$

$$S_r = \sum_{i=1}^{n} (\alpha_i - a_0 - a_1 T_i)^2$$

$$r^2 = \frac{S_t - S_r}{S_t}$$

$S_t - S_r$

α : μin/in/°F

α : in/in/°F

$\times 10^{-12}$

$0 \le r^2 \le 1$

Calculation of S_t

T_i	α_i	$\alpha_i - \bar{\alpha}$
-340	**2.45**	**-2.2250**
-260	**3.58**	**-1.0950**
-180	**4.52**	**-0.15500**
-100	**5.28**	**0.60500**
-20	**5.86**	**1.1850**
60	**6.36**	**1.6850**

$\bar{\alpha} = 4.6750$

$S_t = 10.783$

$\bar{\alpha} = \frac{2.45 + \cdots + 6.36}{6}$

$= 4.6750$

$S_t = (-2.2250)^2$

$+ \cdots$

$+ \cdots$

$+ \cdots$

$+ (1.6850)^2$

$= 10.783$

Calculation of S_r

$$\alpha(T) = 6.0325 + 0.0096964T$$

T_i	Observed α_i	Predicted $a_0 + a_1 T_i$	Residual $\alpha_i - a_0 - a_1 T_i$
-340	**2.45**	**2.7357**	**-0.28571**
-260	**3.58**	**3.5114**	**0.068571**
-180	**4.52**	**4.2871**	**0.23286**
-100	**5.28**	**5.0629**	**0.21714**
-20	**5.86**	**5.8386**	**0.021429**
60	**6.36**	**6.6143**	**-0.25429**

$$S_r = 0.25283$$

Coefficient of determination

$$r^2 = \frac{S_t - S_r}{S_t}$$

$$= \frac{10.783 - 0.25283}{10.783}$$

$$= 0.97655$$

$\simeq 0.98$

Limits of Coefficient of Determination

$$r^2 = \frac{S_t - S_r}{S_t}$$

$$0 \le r^2 \le 1$$

$r^2 = 0$ if $S_t = S_r$

$r^2 = 1$ if $S_r = 0$

$$S_r = \sum_{i=1}^{n} (\alpha_i - a_0 - a_1 T_i)^2$$

$$S_r = \sum_{i=1}^{n} (\alpha_i - a_0 - a_1 T_i)^2$$

$$S_t = \sum_{i=1}^{n} (\alpha_i - \bar{\alpha})^2 = \sum_{i=1}^{n} (\alpha_i - \bar{\alpha} - 0 T_i)^2$$

If $S_t = S_r$

$a_0 = \bar{\alpha}$

$a_1 = 0$

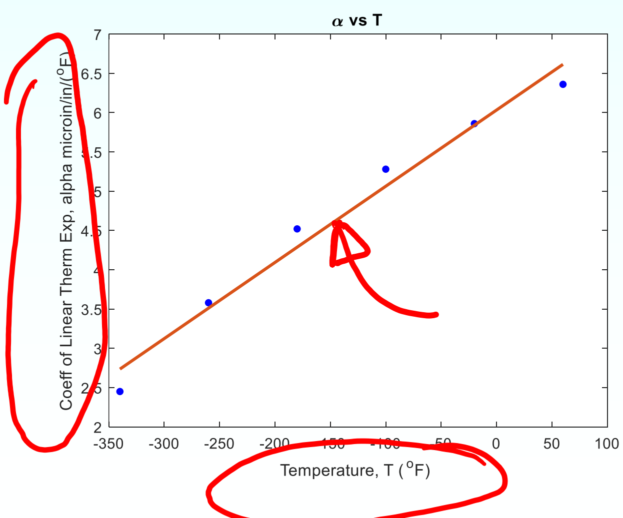
Correlation coefficient

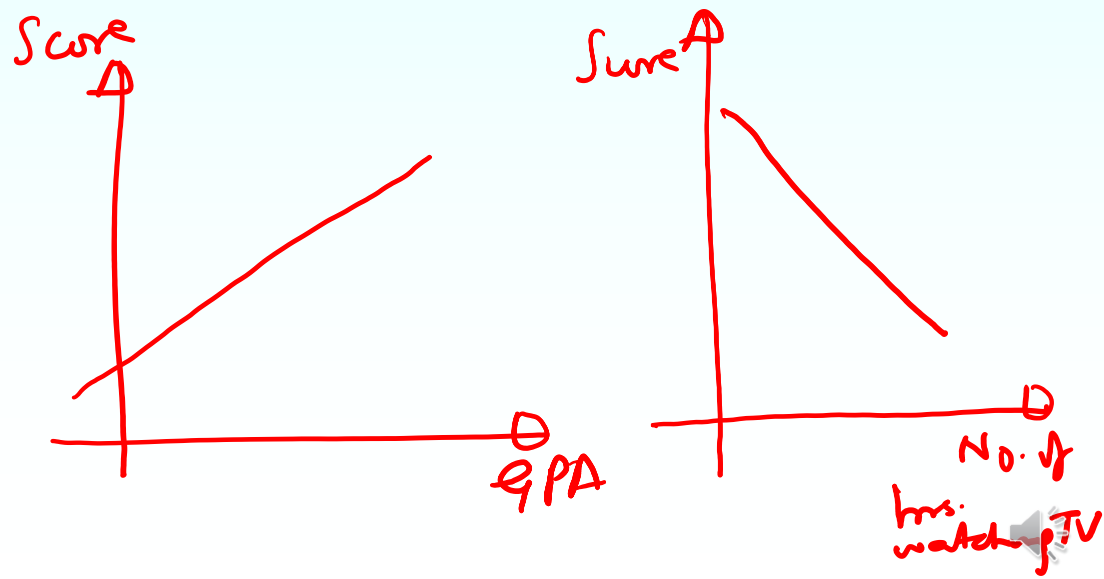
$$r = \sqrt{\frac{S_t - S_r}{S_t}}$$

$$= \sqrt{0.97655}$$

$$= 0.98820$$

How do you know if r is positive or negative ?





What does a particular value of |r| mean?

$r = \pm 1$

0.8 to 1.0 - Very strong relationship

0.6 to 0.8 - Strong relationship

0.4 to 0.6 - Moderate relationship

0.2 to 0.4 - Weak relationship

0.0 to 0.2 - Weak or no relationship

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