Composite Trapezoidal Rule: Motivation and Derivation

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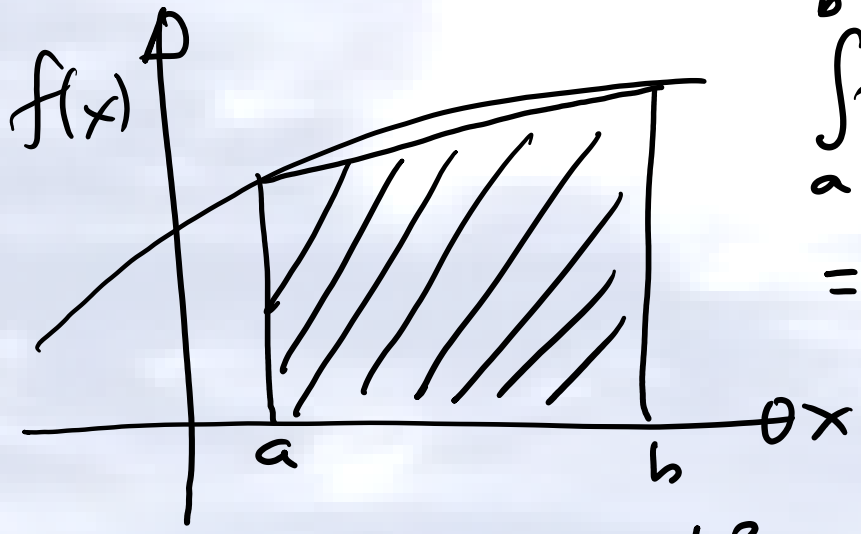
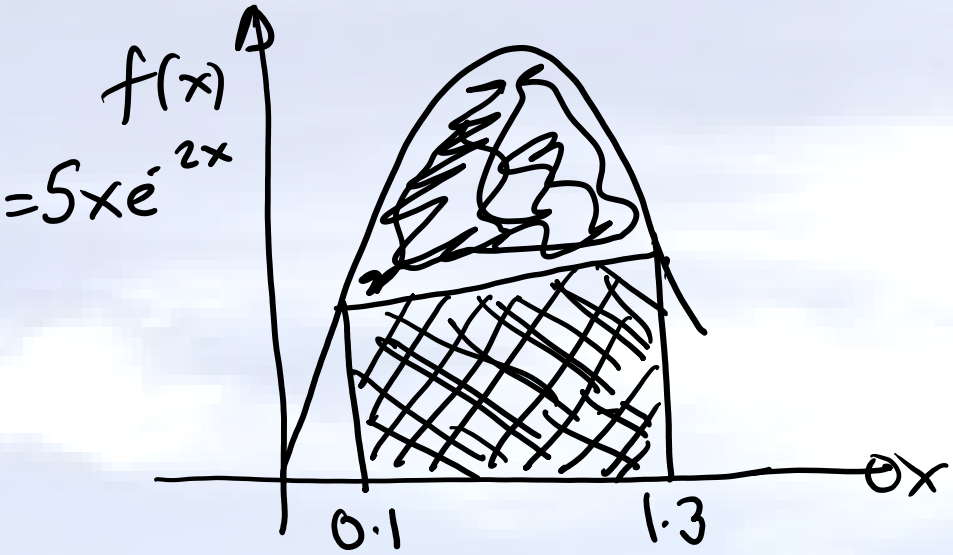


$$\int_a^b f(x)\,dx = (b-a)\left(\frac{f(a)+f(b)}{2}\right)$$

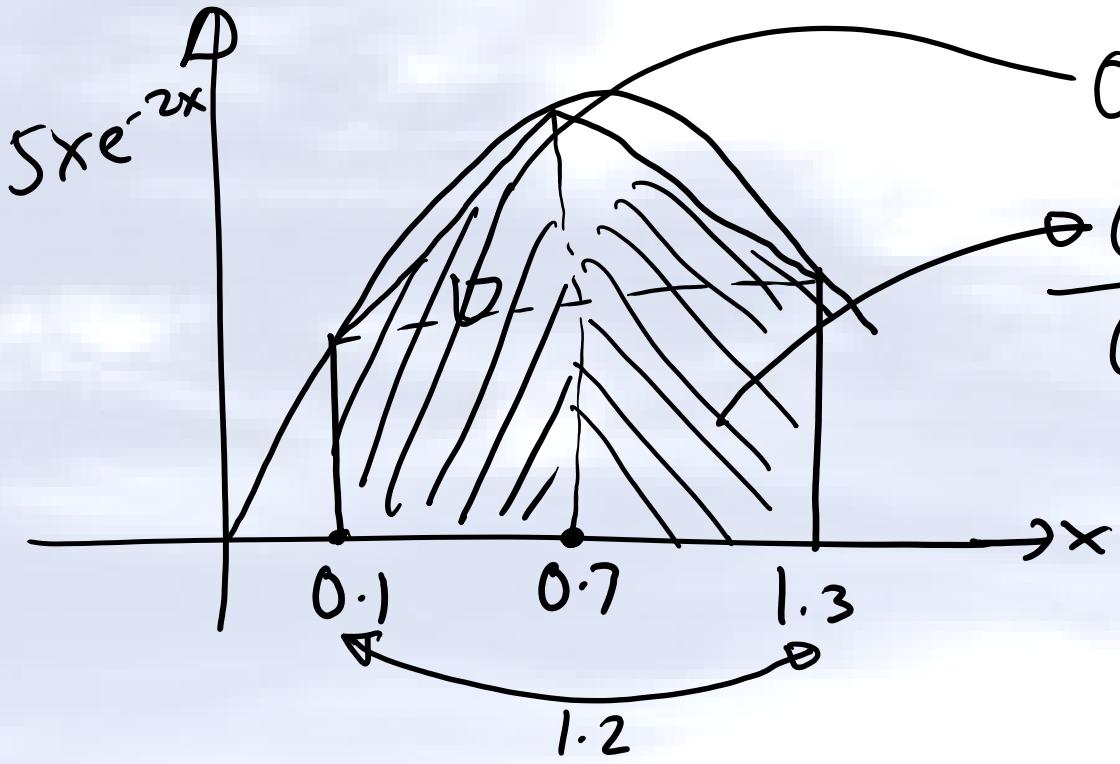


$$\int_{0.1}^{1.3} 5xe^{-2x}\,dx$$

$$\simeq 0.53530$$

$$\text{Exact} = 0.89381$$

$$|\epsilon_t| \simeq 40\%$$



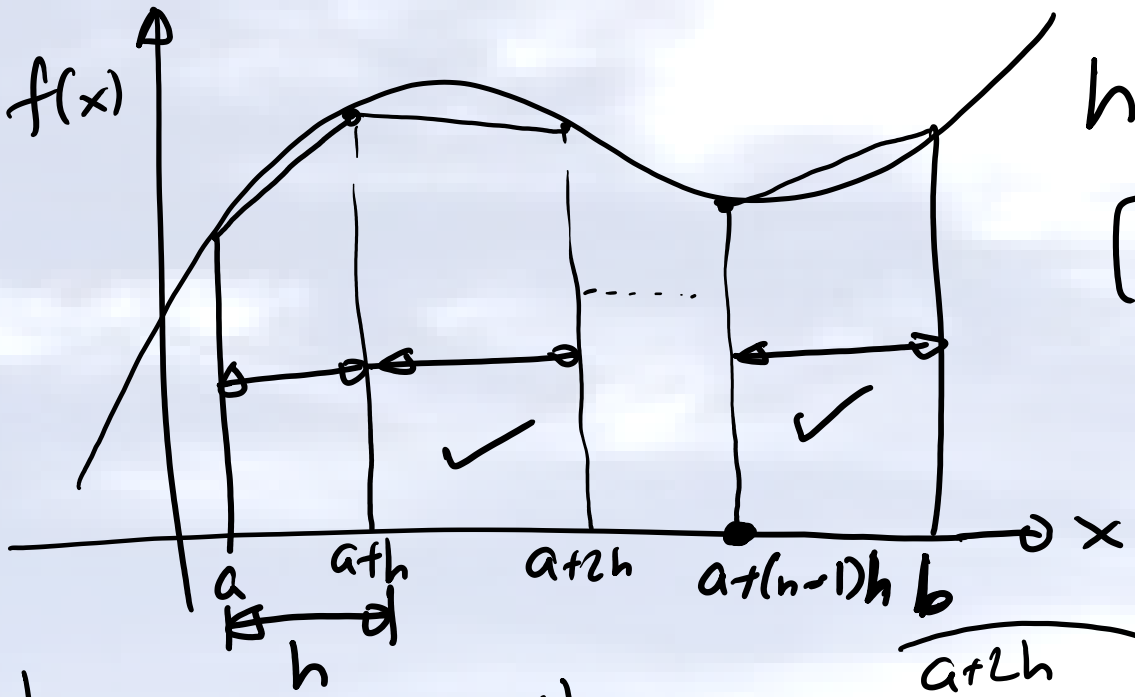
0.38175

0.40377

0.78552

Exact = 0.89387

$|\epsilon_t| \simeq 12\%$



$$h = \frac{b-a}{n}$$

$$\boxed{b = a+nh}$$

$$\int_a^b f(x)\,dx = \int_a^{a+h} f(x)\,dx + \int_{a+h}^{a+2h} f(x)\,dx + \cdots + \int_{a+(n-2)}^{a+(n-1)h} f(x)\,dx + \int_{a+(n-1)h}^{b} f(x)\,dx$$

$$= \underbrace{[(a+h)-a]}_{h}\left(\frac{f(a)+f(a+h)}{2}\right)$$

$$+ \underbrace{((a+2h)-(a+h))}_{h}\left(\frac{f(a+h)+f(a+2h)}{2}\right)$$

$$+ \cdots$$

$$+ \underbrace{((a+(n-1)h)-(a+(n-2)h))}_{h}\left(\frac{f(a+(n-2)h)+f(a+(n-1)h)}{2}\right)$$

$$+ \underbrace{(b-(a+(n-1)h))}_{h}\left(\frac{f(a+(n-1)h)+f(b)}{2}\right)$$

$$= \frac{h}{2}\Big(f(a) + f(a+h) + f(a+h) + f(a+2h) + \cdots + f(a+(n-2)h) + f(a+(n-1)h) + f(a+(n-1)h) + f(b)\Big]$$

$$= \frac{h}{2}\Big(f(a) + \boxed{2f(a+h) + 2f(a+2h) + \cdots + 2f(a+(n-2)h) + 2f(a+(n-1)h)} + f(b)\Big]$$

$$= \frac{h}{2}\left[f(a) + 2\sum_{i=1}^{n-1} f(a+ih) + f(b)\right]$$

$$h = \frac{b-a}{n}$$

$$\int_a^b f(x)\,dx \simeq \frac{b-a}{2n}\left[f(a) + 2\sum_{i=1}^{n-1} f(a+ih) + f(b)\right]$$

1 $\quad$ $2(n-1)$ $\quad$ 1

END

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