

Exact Solution of a First Order ODE: Example



MathForCollege.com
Open Education Resources

<http://nm.MathForCollege.com>

Transforming Numerical Methods Education for STEM Undergraduates



For more details on this topic

- Go to <http://nm.MathForCollege.com>
- Click on Primer for Ordinary Differential Equations



Example of first order ODE exact Solution

Solve the following ordinary differential equation exactly

$$\frac{dy}{dx} + 0.4y = 3e^{-x}, y(0) = 5$$

to find $y(3)$

$$\frac{dy}{dx} + 0.4y = 3e^{-x}, y(0) = 5$$

$$y = \underline{\underline{y_H}} + y_P$$

$$y_H: D = \frac{d}{dx}$$

$$\ominus Dy + 0.4y = 3e^{-x}$$

$$\underline{\underline{(D+0.4)}} y = 3e^{-x}$$

$$m + 0.4 = 0$$

$$m = -0.4$$

$$\boxed{y_H = Ke^{-0.4x}}$$



$$y_p = Ae^{-x}$$

$$\frac{dy_p}{dx} + 0.4y_p = 3e^{-x}$$

$$\frac{d}{dx}(Ae^{-x}) + 0.4(Ae^{-x}) = 3e^{-x}$$

$$-Ae^{-x} + 0.4Ae^{-x} = 3e^{-x}$$

$$-0.6Ae^{-x} = 3e^{-x}$$

$$A = -5$$

$$y_p = -5e^{-x}$$



$$y = y_H + y_p$$

$$= \underline{K e^{-0.4x} - 5e^{-x}}$$

$$y(0) = 5$$

$$5 = K e^{-0.4(0)} - 5e^{-(0)}$$

$$5 = K - 5$$

$$K = 10$$

$$\boxed{y = 10e^{-0.4x} - 5e^{-x}}$$

$$y(3) = 10e^{-0.4(3)} - 5e^{-3}$$

$$= \boxed{2.7630}$$

\boxed{EWD}



You are free

- to **Share** – to copy, distribute, display and perform the work
- to **Remix** – to make derivative works

under the following conditions

- **Attribution** — You must attribute the work in the manner specified by the author or licensor (but not in any way that suggests that they endorse you or your use of the work).
- **Noncommercial** — You may not use this work for commercial purposes.
- **Share Alike** — If you alter, transform, or build upon this work, you may distribute the resulting work only under the same or similar license to this one.



Acknowledgement

This instructional resource is brought to you by
Numerical Methods for STEM undergraduate

<http://nm.MathForCollege.com>

Committed to bringing numerical methods to the
undergraduate

This material is based upon work supported by the National Science Foundation under Grant #2013271 (Transforming Undergraduate Engineering Education through Adaptive Learning and Student Data Analytics). Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the National Science Foundation.





MathForCollege.com

Open Education Resources

Like what you see - tell your friends.

Subscribe to the NumericalMethodsGuy Channel – help us reach our goal of 100,000 subscribers.

