

# Solving Second-Order Ordinary Differential Equations Exactly: Distinct Roots



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# Problem Statement

Solve the ordinary differential equation exactly

$$\left(1 \frac{d^2 y}{dx^2} + 5 \frac{dy}{dx} + 6y\right) = \underline{\underline{7e^{-4x}}}, \quad \underline{y(0) = 13}, \quad \underline{\underline{\frac{dy}{dx}(0) = 17}}$$

$$y = y_H + y_P$$

$$y_H: 1m^2 + 5m + 6 = 0$$

$$m^2 + 5m + 6 = 0$$

$$(m+3)(m+2) = 0$$

$$m = -3, -2$$

$$y_H = K_1 e^{m_1 x} + K_2 e^{m_2 x}$$

$$= K_1 e^{-3x} + K_2 e^{-2x}$$



$$\frac{d^2 y}{dx^2} + 5 \frac{dy}{dx} + 6y = 7e^{-4x}$$

$$y_H = \frac{K_1 e^{-3x} + K_2 e^{-2x}}{\quad}$$

$$y_P = \boxed{Ae^{-4x}}$$

$$\frac{d^2}{dx^2} (Ae^{-4x}) + 5 \frac{d}{dx} (Ae^{-4x}) + 6(Ae^{-4x}) = 7e^{-4x}$$

$$\underline{16} Ae^{-4x} + 5 \underline{(-4Ae^{-4x})} + \underline{6(Ae^{-4x})} = 7e^{-4x}$$

$$2Ae^{-4x} = 7e^{-4x}$$

$$A = 3.5$$

$$y_P = Ae^{-4x} \\ = 3.5e^{-4x}$$



$$y_H = K_1 e^{-3x} + K_2 e^{-2x}$$

$$y_p = 3.5 e^{-4x}$$

$$y = y_H + y_p$$

$$= \underline{\underline{K_1}} e^{-3x} + \underline{\underline{K_2}} e^{-2x} + 3.5 e^{-4x}$$

$$y(0) = 13$$

$$13 = K_1 e^{-3(0)} + K_2 e^{-2(0)} + 3.5 e^{-4(0)}$$

$$= K_1 + K_2 + \underline{\underline{3.5}}$$

$$\boxed{K_1 + K_2 = 9.5}$$

①



$$\frac{dy}{dx}(0) = 17$$

$$y = K_1 e^{-3x} + K_2 e^{-2x} + 3.5 e^{-4x}$$

$$\frac{dy}{dx} = -3K_1 e^{-3x} - 2K_2 e^{-2x} - 14e^{-4x}$$

$$\frac{dy}{dx}(0) = 17$$

$$17 = -3K_1 e^{-3(0)} - 2K_2 e^{-2(0)} - 14e^{-4(0)}$$

$$17 = -3K_1 - 2K_2 - 14$$

$$3K_1 + 2K_2 = -17 - 14 = -31 \quad \text{--- (2)}$$



$$3K_1 + 2K_2 = -31 \quad \text{---} \quad \textcircled{2}$$

$$(K_1 + K_2 = 9.5) * 3 \quad \text{---} \quad \textcircled{1}$$

$$3K_1 + 3K_2 = 28.5 \quad \text{---} \quad \textcircled{3}$$

$$\underline{\text{Eqn } \textcircled{2} - \text{Eqn } \textcircled{3}}$$

$$-K_2 = -59.5$$

$$K_2 = 59.5$$

Substitute  $K_2$  in Eqn  $\textcircled{1}$ ,

$$K_1 + K_2 = 9.5$$

$$K_1 + (59.5) = 9.5$$

$$K_1 = -50$$

$$y = K_1 e^{-3x} + K_2 e^{-2x} + 3.5 e^{-4x}$$

$$y = -50 e^{-3x} + 59.5 e^{-2x} + 3.5 e^{-4x}$$

END

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# Acknowledgement

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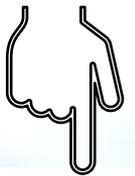
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