

Exact Solution of a 2nd Order ODE – (Identical Roots of Characteristic Equation)



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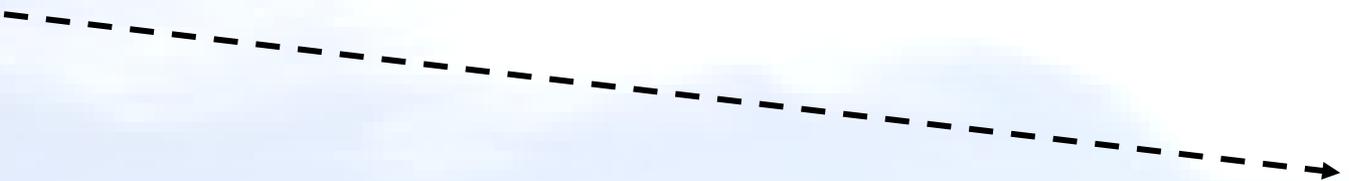
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$$y'' + 4y' + 4y = 10t,$$
$$y(0) = 5, \quad y'(0) = 2$$

$$y = y_H + y_P$$

Homogeneous

$$\textcircled{\ominus} \quad m^2 + 4m + 4 = 0$$

$$(m+2)^2 = 0 \Rightarrow m = -2, -2$$

$$m = \frac{-4 \pm \sqrt{4^2 - 4(1)(4)}}{2(1)}$$
$$= -2, -2$$



$$m = -2, -2$$

$$y_{1+} = \underline{K_1 e^{-2t}} + \underline{K_2 t e^{-2t}}$$

Particular part

$$y_p = \underline{A}t + \underline{B}$$

$$y'' + 4y' + 4y = 10t$$

$$\frac{d^2}{dt^2} (At + B) + 4 \frac{d}{dt} (At + B) + 4(At + B) = 10t$$

$$0 + (4A) + (\underline{4At} + 4B) = 10t$$

$$\underline{(4A)}t + (\underline{4A + 4B}) = \underline{10t} + \underline{0}$$



$$4A = 10 \quad \text{--- (1)}$$

$$4A + 4B = 0 \quad \text{--- (2)}$$

$$\ominus \quad \underline{A = +2.5}$$

$$\ominus \quad 4(2.5) + 4B = 0$$

$$\underline{B = -2.5}$$

$$\underline{y_p = 2.5t - 2.5}$$

$$y = y_h + y_p$$

$$= \underline{\underline{K_1}} e^{-2t} + \underline{\underline{K_2}} t e^{-2t} + 2.5t - 2.5$$



$$y(0) = 5$$

$$5 = K_1 e^{-2(0)} + K_2(0) e^{-2(0)} + 2.5(0) - 2.5$$

$$5 = K_1 + 0 + 0 - 2.5$$

$$K_1 = 7.5$$

$$y'(0) = 2$$

$$y' = \frac{d}{dt} (K_1 e^{-2t} + \underline{\underline{K_2 t e^{-2t}}} + \frac{2.5t}{\underline{\underline{-2.5}}})$$

$$= \underline{\underline{-2K_1 e^{-2t} + K_2 e^{-2t} - 2K_2 t e^{-2t} + 2.5 - 0}}$$



$$y' = -2K_1 e^{-2t} + K_2 e^{-2t} - 2K_2 t e^{-2t} + 2.5$$

$$y'(0) = 2$$

$$2 = -2K_1 e^{-2(0)} + K_2 e^{-2(0)} - 2K_2(0)e^{-2(0)} + 2.5$$

$$2 = -2K_1 + K_2 + 0 + 2.5$$

$$2 = -2(7.5) + K_2 + 2.5$$

$$K_2 = 14.5$$

$$y = K_1 e^{-2t} + K_2 t e^{-2t} + 2.5t - 2.5$$

$$y = 7.5 e^{-2t} + 14.5 t e^{-2t} + 2.5t - 2.5$$

END



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