# Runge-Kutta Second Order Method of Solving ODEs - Theory

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$$\frac{dy}{dx} = f(x,y), \quad y(x_0) = y_0$$

$$y_{i+1} = y_i + \left.\frac{dy}{dx}\right|_{x_i,y_i}(x_{i+1}-x_i) + \frac{1}{2!}\left.\frac{d^2y}{dx^2}\right|_{x_i,y_i}(x_{i+1}-x_i)^2 + \frac{1}{3!}\left.\frac{d^3y}{dx^3}\right|_{x_i,y_i}(x_{i+1}-x_i)^3 + \cdots$$

$$\boxed{y_{i+1} = y_i + f(x_i,y_i)\,h} + \boxed{\frac{1}{2!}f'(x_i,y_i)\,h^2} + \frac{1}{3!}f''(x_i,y_i)\,h^3 + \cdots$$

$$\frac{dy}{dx} = 5e^{-x} - 3y, \quad y(0) = 5$$

$$= f(x,y)$$

$$f'(x,y) = \frac{\partial f}{\partial x} + \frac{\partial f}{\partial y}\frac{dy}{dx}$$

$$= -5e^{-x} + (-3)(5e^{-x} - 3y)$$

$$y_{i+1} = y_i + (a_1 k_1 + a_2 k_2)h$$

$$k_1 = f(x_i, y_i)$$

$$k_2 = f(x_i + p_1 h, y_i + q_{11} k_1 h)$$

$a_1, a_2, p_1, q_{11}$

$$y_{i+1} = y_i + f(x_i, y_i)h + \frac{1}{2!} f''(x_i, y_i)h^2 + O(h^3)$$

$$a_1 + a_2 = 1$$

$$a_2 p_1 = \frac{1}{2}$$

$$a_2 q_{11} = \frac{1}{2}$$

# Heun's method $(a_2 \equiv \frac{1}{2})$

$$a_1 + a_2 = 1, \quad a_2 p_1 = \frac{1}{2}, \quad a_2 q_{11} = \frac{1}{2}$$

$$a_1 = \frac{1}{2} \qquad p_1 = 1 \qquad q_{11} = 1$$

$$a_1 = \frac{1}{2}, \; a_2 = \frac{1}{2}, \; p_1 = 1, \; q_{11} = 1$$

$$y_{i+1} = y_i + \left(\frac{1}{2}k_1 + \frac{1}{2}k_2\right)h$$

$$k_1 = f(x_i, y_i)$$

$$k_2 = f(x_i + h, y_i + k_1 h)$$

Heun's method

Ralston's method ($a_2 = \frac{2}{3}$)

Mid-pt. method ($a_2 = 1$)

END

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# Acknowledgement

This instructional resource is brought to you by Numerical Methods for STEM undergraduate
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This material is based upon work supported by the National Science Foundation under Grant #2013271 (Transforming Undergraduate Engineering Education through Adaptive Learning and Student Data Analytics). Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the National Science Foundation.

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