Runge-Kutta Second Order Method of Solving ODEs - Applications

MathForCollege.com
Open Education Resources

http://nm.MathForCollege.com
Transforming Numerical Methods Education for STEM Undergraduates

For more resources and playlist on this topic

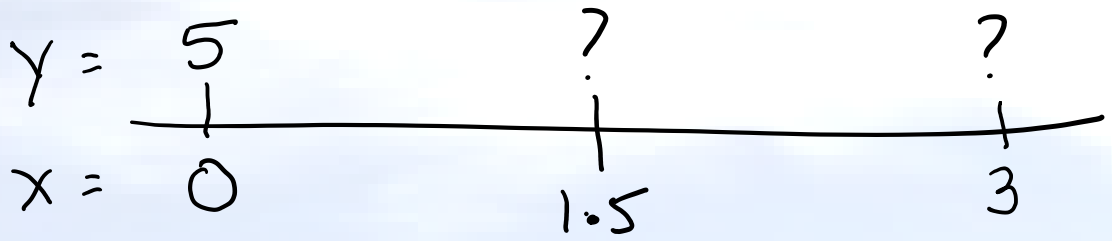
- Go to http://nm.MathForCollege.com
- Click on Runge-Kutta Method

Problem statement

Given

$$\frac{dy}{dx} + 0.4y = 3e^{-x}, \quad y(0) = 5$$

Use a Runge-Kutta 2nd order method called Heun's method to find $y(3)$. Choose a step size of $h = 1.5$.



$$\frac{dy}{dx} + 0.4y = 3e^{-x}, \quad y(0) = 5$$

$$\frac{dy}{dx} = 3e^{-x} - 0.4y = f(x, y)$$

$$y_{i+1} = y_i + \left(\frac{1}{2}k_1 + \frac{1}{2}k_2\right)h$$

$$k_1 = f(x_i, y_i)$$

$$k_2 = f(x_i + h, y_i + k_1 h)$$

$$i=0, \quad x_0=0, \quad y_0=5$$

$$k_1 = f(x_0, y_0) = f(0,5)$$

$$= 3e^{-0} - 0.4(5) = \boxed{1}$$

$$k_2 = f(x_0+h, y_0+k_1 h)$$

$$= f(0+1.5,\ 5+(1)(1.5)) = f(1.5, 6.5)$$

$$= 3e^{-1.5} - 0.4(6.5) = -1.93061$$

$$y_1 = y_0 + \left(\frac{1}{2}k_1 + \frac{1}{2}k_2\right)h$$

$$= 5 + \left(\frac{1}{2}(1) + \frac{1}{2}(-1.93061)\right) * 1.5$$

$$= 4.30204 \simeq y(x_1) \simeq y(1.5)$$

$\underline{i=1}$, $x_1 = 1.5$, $y_1 = 4.30204$

$$k_1 = f(x_1, y_1) = f(1.5, 4.30204)$$

$$= 3e^{-1.5} - 0.4(4.30204)$$

$$= -1.05143$$

$$k_2 = f(x_1 + h, y_1 + k_1 h)$$

$$= f(1.5 + 1.5, 4.30204 + (-1.05143)(1.5))$$

$$= f(3, 2.7249) = -0.9406$$

$$= 3e^{-3} - 0.4(2.7249)$$

$$y_2 = y_1 + \left(\frac{1}{2}k_1 + \frac{1}{2}k_2\right)h$$

$$= 4.30204 + \left(\frac{1}{2}(-1.05143) + \frac{1}{2}(-0.9406)\right) \times 1.5$$

$$= \boxed{2.8082} \simeq y(x=3)$$

END

You are free

- to **Share** – to copy, distribute, display and perform the work
- to **Remix** – to make derivative works

under the following conditions

- **Attribution** — You must attribute the work in the manner specified by the author or licensor (but not in any way that suggests that they endorse you or your use of the work).
- **Noncommercial** — You may not use this work for commercial purposes.
- **Share Alike** — If you alter, transform, or build upon this work, you may distribute the resulting work only under the same or similar license to this one.

Acknowledgement

This instructional resource is brought to you by Numerical Methods for STEM undergraduate
http://nm.MathForCollege.com
Committed to bringing numerical methods to the undergraduate

This material is based upon work supported by the National Science Foundation under Grant #2013271 (Transforming Undergraduate Engineering Education through Adaptive Learning and Student Data Analytics). Any opinions, findings, and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the National Science Foundation.

MathForCollege.com

Open Education Resources

Like what you see - tell your friends.

Subscribe to the NumericalMethodsGuy Channel – help us reach our goal of 100,000 subscribers.