Trapezoidal Rule: Derivation via Method of Undetermined Coefficients

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$$\int_a^b f(x)\,dx \simeq (b-a)\left(\frac{f(a)+f(b)}{2}\right)$$

$$\int_a^b f(x)\,dx \simeq C_1 f(a) + C_2 f(b)$$

Let the formula be exact

for $f(x) = a_0 + a_1 x$

$$\int_a^b f(x)\,dx \simeq c_1 f(a) + c_2 f(b)$$

$$f(x) = a_0 + a_1 x$$

$$\int_a^b (a_0 + a_1 x)\,dx = \left[a_0 x + a_1 \frac{x^2}{2}\right]_a^b$$

$$= \left[a_0 (b-a) + a_1 \frac{b^2 - a^2}{2}\right]$$

$$c_1 f(a) + c_2 f(b)$$

$$= c_1 (a_0 + a_1 a) + c_2 (a_0 + a_1 b)$$

$$a_0(b-a) + a_1\left(\frac{b^2-a^2}{2}\right) = c_1(a_0+a_1a) + c_2(a_0+a_1b)$$

$$= c_1a_0 + c_1a_1a + c_2a_0 + c_2a_1b$$

$$= a_0(c_1+c_2) + a_1(c_1a+c_2b)$$

$$c_1 + c_2 = b - a \quad \text{---} (1)$$

$$c_1a + c_2b = \frac{b^2-a^2}{2} \quad \text{---} (2)$$

$(c_1 + c_2 = b - a) * a$

$c_1 a + c_2 b = \frac{b^2 - a^2}{2}$ ✓

$c_1 a + c_2 a = a(b-a)$ ✓

$c_2 b - c_2 a = \frac{b^2 - a^2}{2} - a(b-a)$

$c_2 (b-a) = \frac{(b-a)(b+a)}{2} - a(b-a)$

$c_2 = \frac{b+a}{2} - a = \frac{b-a}{2}$

$c_1 + c_2 = b - a$

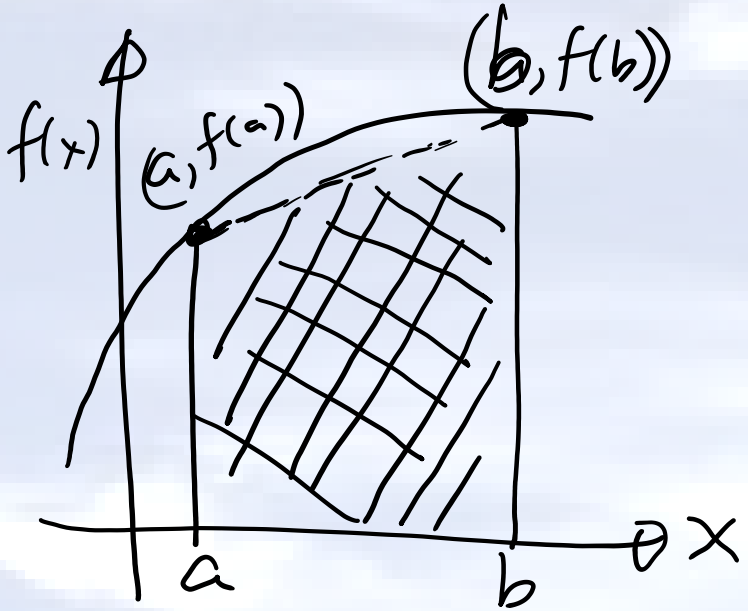
$\frac{b-a}{2}$

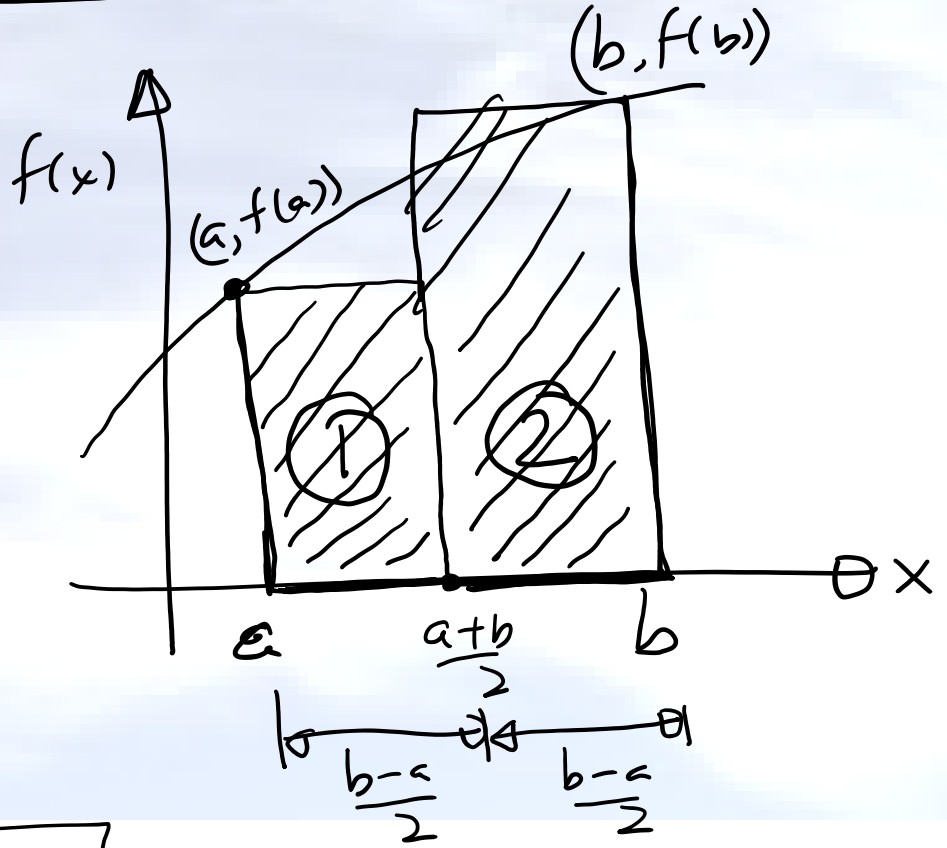
$c_1 = \frac{b-a}{2}$

$$c_1 = \frac{b-a}{2}, \quad c_2 = \frac{b-a}{2}$$

$$\int_a^b f(x)\,dx \simeq c_1 f(a) + c_2 f(b) = \boxed{\frac{b-a}{2} f(a)} + \boxed{\frac{b-a}{2} f(b)}$$

$$= (b-a)\left(\frac{f(a)+f(b)}{2}\right)$$





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