One-Point Gauss Quadrature Rule

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$$\int_a^b f(x)\,dx \simeq c_1 f(x_1), \quad a \le x_1 \le b$$

Let $f(x) = a_0 + a_1 x$

$$\int_a^b (a_0 + a_1 x)\,dx = \left[a_0 x + a_1 \frac{x^2}{2}\right]_a^b$$

$$= a_0(b-a) + a_1 \frac{b^2 - a^2}{2}$$

$$c_1 f(x_1) = c_1 (a_0 + a_1 x_1)$$

$$a_0(b-a) + a\left(\frac{b^2-a^2}{2}\right) = c_1(a_0 + a_1 x_1)$$

$$= c_1 a_0 + c_1 a_1 x_1$$

$$= a_0(c_1) + a_1(c_1 x_1)$$

$$c_1 = b - a \quad \text{---} (1)$$

$$c_1 x_1 = \frac{b^2-a^2}{2} \quad \text{---} (2)$$

$$(b-a)\, x_1 = \frac{b^2-a^2}{2}$$

$$(b-a)\, x_1 = \frac{(b-a)(b+a)}{2}$$

$$x_1 = \frac{b+a}{2}$$

$$c_1 = (b-a), \quad x_1 = \frac{b+a}{2}$$

$$\int_a^b f(x)\,dx \simeq c_1 f(x_1)$$

$$= (b-a) f\left(\frac{b+a}{2}\right)$$

Example of One-Point Gauss Quad

The following integral is given

$$\int_{0.1}^{1.3} 5xe^{-2x}dx$$

a) Use the one-point Gaussian quadrature rule to estimate the value of the integral.

b) Find the true error for part (a).

c) Find the absolute relative true error, $|\epsilon_t|$, for part (a).

$$\int_a^b f(x)\,dx \simeq (b-a)\, f\left(\frac{a+b}{2}\right)$$

a) $$\int_{0.1}^{1.3} f(x)\,dx \simeq (1.3-0.1)\, f\left(\frac{1.3+0.1}{2}\right)$$

$$= 1.2 f(0.6)$$

$$= 1.2\left[5(0.6)e^{-2(0.6)}\right]$$

$$= 1.0357$$

b) E_t = True value - Approx. value

$= 0.89386 - 1.0357$

$= -0.14183$

c) $|\epsilon_t| = \left| \frac{E_t}{\text{True value}} \right| * 100$

$= \left| \frac{-0.14183}{0.89386} \right| * 100$

$= 15.86\%$

Trap Rule

$$\int_a^b f(x)\,dx \simeq (b-a)\,\frac{f(c)+f(b)}{2}$$

$$\int_a^b f(x)\,dx \simeq (b-a)\,f\left(\frac{a+b}{2}\right)$$

END

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