Chapter 03.04
Newton-Raphson Method of Solving a Nonlinear Equation – More Examples
Chemical Engineering

Example 1

You have a spherical storage tank containing oil. The tank has a diameter of $6\,\text{ft}$. You are asked to calculate the height h to which a dipstick 8 ft long would be wet with oil when immersed in the tank when it contains $6\,\text{ft}^3$ of oil.

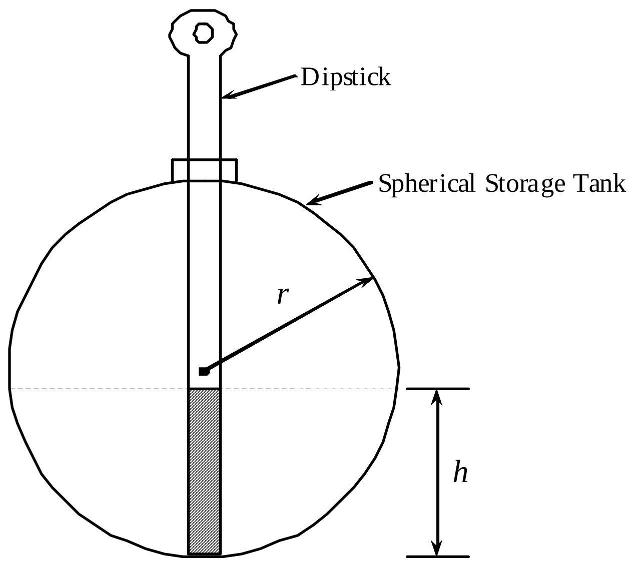

Figure 1 Spherical storage tank problem.

The equation that gives the height h of the liquid in the spherical tank for the given volume and radius is given by

$$f(h) = h^3 - 9h^2 + 3.8197 = 0$$

Use the Newton-Raphson method of finding roots of equations to find the height h to which the dipstick is wet with oil. Conduct three iterations to estimate the root of the above equation. Find the absolute relative approximate error at the end of each iteration and the number of significant digits at least correct at the end of each iteration.

Solution

$$f(h) = h^3 - 9h^2 + 3.8197$$
$$f'(h) = 3h^2 - 18h$$

Let us take the initial guess of the root of $f(h) = 0$ as $h_0 = 1$.

<u>Iteration 1</u>

The estimate of the root is

$$\begin{aligned} h_1 &= h_0 - \frac{f(h_0)}{f'(h_0)} \\ &= 1 - \frac{(1)^3 - 9(1)^2 + 3.8197}{3(1)^2 - 18(1)} \\ &= 1 - \frac{-4.1803}{-15} \\ &= 1 - (0.27869) \\ &= 0.72131 \end{aligned}$$

The absolute relative approximate error $|\in_a|$ at the end of Iteration 1 is

$$\begin{aligned} |\in_a| &= \left|\frac{h_1 - h_0}{h_1}\right| \times 100 \\ &= \left|\frac{0.72131 - 1}{0.72131}\right| \times 100 \\ &= 38.636\% \end{aligned}$$

The number of significant digits at least correct is 0, as you need an absolute relative approximate error of 5% or less for one significant digit to be correct in your result.

<u>Iteration 2</u>

The estimate of the root is

$$\begin{aligned} h_2 &= h_1 - \frac{f(h_1)}{f'(h_1)} \\ &= 0.72131 - \frac{(0.72131)^3 - 9(0.72131)^2 + 3.8197}{3(0.72131)^2 - 18(0.72131)} \\ &= 0.72131 - \frac{-0.48764}{-11.423} \\ &= 0.72131 - (0.042690) \\ &= 0.67862 \end{aligned}$$

The absolute relative approximate error $|\in_a|$ at the end of Iteration 2 is

$$|\in_a| = \left|\frac{h_2 - h_1}{h_2}\right| \times 100$$

$$= \left|\frac{0.67862 - 0.72131}{0.67862}\right| \times 100$$
$$= 6.2907\%$$

The number of significant digits at least correct is 0.

<u>Iteration 3</u>

The estimate of the root is

$$h_3 = h_2 - \frac{f(h_2)}{f'(h_2)}$$
$$= 0.67862 - \frac{(0.67862)^3 - 9(0.67862)^2 + 3.8197}{3(0.67862)^2 - 18(0.67862)}$$
$$= 0.67862 - \frac{-0.012536}{-10.834}$$
$$= 0.67862 - (0.0011572)$$
$$= 0.67747$$

The absolute relative approximate error $|\in_a|$ at the end of Iteration 3 is

$$|\in_a| = \left|\frac{h_3 - h_2}{h_3}\right| \times 100$$
$$= \left|\frac{0.67747 - 0.67862}{0.67747}\right| \times 100$$
$$= 0.17081\%$$

Hence the number of significant digits at least correct is given by the largest value of m for which

$$|\in_a| \le 0.5 \times 10^{2-m}$$
$$0.17081 \le 0.5 \times 10^{2-m}$$
$$0.34162 \le 10^{2-m}$$
$$\log(0.34162) \le 2 - m$$
$$m \le 2 - \log(0.34162) = 2.4665$$

So

$$m = 2$$

The number of significant digits at least correct in the estimated root 0.67747 is 2.

NONLINEAR EQUATIONS	
Topic	Newton-Raphson Method-More Examples
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