

Chapter 03.04

Newton-Raphson Method of Solving a Nonlinear Equation – More Examples

Computer Science

Example 1

To find the inverse of a number a , one can use the equation

$$f(c) = a - \frac{1}{c} = 0$$

where c is the inverse of a .

Use the Newton-Raphson method of finding roots of equations to find the inverse of $a = 2.5$. Conduct three iterations to estimate the root of the above equation. Find the absolute relative approximate error at the end of each iteration and the number of significant digits at least correct at the end of each iteration.

Solution

$$f(c) = a - \frac{1}{c} = 0$$

$$f'(c) = \frac{1}{c^2}$$

$$\begin{aligned}c_{i+1} &= c_i - \frac{f(c_i)}{f'(c_i)} \\ &= c_i - \frac{a - \frac{1}{c_i}}{\frac{1}{c_i^2}} \\ &= c_i - c_i^2 \left(a - \frac{1}{c_i} \right) \\ &= c_i - c_i^2 a + c_i \\ &= 2c_i - c_i^2 a\end{aligned}$$

Let us take the initial guess of the root of $f(c) = 0$ as $c_0 = 0.5$.

Iteration 1

The estimate of the root is

$$\begin{aligned}c_1 &= 2c_0 - c_0^2 a \\ &= 2(0.5) - (0.5)^2 (2.5) \\ &= 0.375\end{aligned}$$

The absolute relative approximate error $|\epsilon_a|$ at the end of Iteration 1 is

$$\begin{aligned}|\epsilon_a| &= \left| \frac{c_1 - c_0}{c_1} \right| \times 100 \\ &= \left| \frac{0.375 - 0.5}{0.375} \right| \times 100 \\ &= 33.333\%\end{aligned}$$

The number of significant digits at least correct is 0, as you need an absolute relative approximate error of less than 5% for one significant digit to be correct in your result.

Iteration 2

The estimate of the root is

$$\begin{aligned}c_2 &= 2c_1 - c_1^2 a \\ &= 2(0.375) - (0.375)^2 (2.5) \\ &= 0.39844\end{aligned}$$

The absolute relative approximate error $|\epsilon_a|$ at the end of Iteration 2 is

$$\begin{aligned}|\epsilon_a| &= \left| \frac{c_2 - c_1}{c_2} \right| \times 100 \\ &= \left| \frac{0.39844 - 0.375}{0.39844} \right| \times 100 \\ &= 5.8824\%\end{aligned}$$

The number of significant digits at least correct is 0.

Iteration 3

The estimate of the root is

$$\begin{aligned}c_3 &= 2c_2 - c_2^2 a \\ &= 2(0.3984) - (0.3984)^2 (2.5) \\ &= 0.39999\end{aligned}$$

The absolute relative approximate error $|\epsilon_a|$ at the end of Iteration 3 is

$$\begin{aligned}|\epsilon_a| &= \left| \frac{0.39999 - 0.39844}{0.39999} \right| \times 100 \\ &= 0.38911\%\end{aligned}$$

Hence the number of significant digits at least correct is given by the largest value of m for which

$$|\epsilon_a| \leq 0.5 \times 10^{2-m}$$

$$0.38911 \leq 0.5 \times 10^{2-m}$$

$$0.77821 \leq 10^{2-m}$$

$$\log(0.77821) \leq 2 - m$$

$$m \leq 2 - \log(0.77821) = 2.1089$$

So

$$m = 2$$

The number of significant digits at least correct in the estimated root 0.39999 is 2.

NONLINEAR EQUATIONS

Topic	Newton-Raphson Method-More Examples
Summary	Examples of Newton-Raphson Method
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