07.03
Simpson's 1/3 Rule for Integration-More Examples Computer Engineering

Example 1

Human vision has the remarkable ability to infer 3D shapes from 2D images. The intriguing question is: can we replicate some of these abilities on a computer? Yes, it can be done and to do this, integration of vector fields is required. The following integral needs to integrated.

$$I = \int_0^{100} f(x)dx$$

where

$$\begin{aligned} f(x) &= 0,\ 0 < x < 30 \\ &= -9.1688\times10^{-6}x^3 + 2.7961\times10^{-3}x^2 - 2.8487\times10^{-1}x + 9.6778,\ 30 \le x \le 172 \\ &= 0,\ 172 < x < 200 \end{aligned}$$

a) Use Simpson's 1/3 Rule to find the probability.
b) Find the true error, E_t, for part (a).
c) Find the absolute relative true error for part (a).

Solution

a) $$I \approx \frac{b-a}{6}\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right]$$

$$a = 0$$
$$b = 100$$
$$\frac{a+b}{2} = 50$$

$$\begin{aligned} f(x) &= 0,\ 0 < x < 30 \\ &= -9.1688\times10^{-6}x^3 + 2.7961\times10^{-3}x^2 - 2.8487\times10^{-1}x + 9.6778,\ 30 \le x \le 172 \\ &= 0,\ 172 < x < 200 \end{aligned}$$

$$f(0) = 0$$

$$\begin{aligned} f(100) &= -9.1688\times10^{-6}\times(100)^3 + 2.7961\times10^{-3}\times(100)^2 - 2.8487\times10^{-1}\times(100) + 9.6778 \\ &= -0.017000 \end{aligned}$$

$$\begin{aligned} f(50) &= -9.1688\times10^{-6}\times(50)^3 + 2.7961\times10^{-3}\times(50)^2 - 2.8487\times10^{-1}\times(50) + 9.6778 \\ &= 1.2784 \end{aligned}$$

$$I \approx \left(\frac{b-a}{6}\right)\left[f(a)+4f\left(\frac{a+b}{2}\right)+f(b)\right]$$
$$\approx \left(\frac{100-0}{6}\right)\left[f(0)+4f(50)+f(100)\right]$$
$$\approx \left(\frac{100}{6}\right)\left[0+4(1.2784)+(-0.017)\right]$$
$$\approx 84.947$$

b) The exact value of the above integral is found using Maple for calculating the true error and relative true error.

$$I = \int_0^{100} f(x)dx$$
$$= 60.793$$

so the true error is

$$E_t = True\ Value - Approximate\ Value$$
$$= 60.793 - (84.947)$$
$$= -24.154$$

c) The absolute relative true error, $|\in_t|$, would then be

$$|\in_t| = \left|\frac{\text{True Error}}{\text{True Value}}\right| \times 100\,\%$$
$$= \left|\frac{60.793-(84.947)}{60.793}\right| \times 100\,\%$$
$$= 39.732\,\%$$

Example 2

Human vision has the remarkable ability to infer 3D shapes from 2D images. The intriguing question is: can we replicate some of these abilities on a computer? Yes, it can be done and to do this, integration of vector fields is required. The following integral needs to integrated.

$$I = \int_0^{100} f(x)dx$$

where

$$f(x) = 0,\ 0 < x < 30$$
$$= -9.1688\times10^{-6}x^3 + 2.7961\times10^{-3}x^2 - 2.8487\times10^{-1}x + 9.6778,\ \ 30 \le x \le 172$$
$$= 0,\ \ 172 < x < 200$$

a) Use four segment Simpson's 1/3 Rule to find the value of the integral
b) Find the true error, E_t, for part (a).
c) Find the absolute relative true error for part (a).

Solution

a) Using n segment Simpson's 1/3 Rule,

$$I \approx \frac{b-a}{3n}\left[f(x_0)+4\sum_{\substack{i=1\\i=odd}}^{n-1} f(x_i)+2\sum_{\substack{i=2\\i=even}}^{n-2} f(x_i)+f(x_n)\right]$$

$$n = 4$$
$$a = 0$$
$$b = 100$$
$$h = \frac{b-a}{n} = \frac{100-0}{4} = 25$$

$$\begin{aligned} f(x) &= 0,\ 0 < x < 30 \\ &= -9.1688\times10^{-6}x^3 + 2.7961\times10^{-3}x^2 - 2.8487\times10^{-1}x + 9.6778,\ 30 \le x \le 172 \\ &= 0,\ 172 < x < 200 \end{aligned}$$

So

$$f(x_0) = f(0)$$
$$f(0) = 0$$

$$f(x_1) = f(0+25) = f(25)$$
$$f(25) = 0$$

$$f(x_2) = f(25+25) = f(50)$$
$$f(50) = -9.1688\times10^{-6}\times(50)^3 + 2.7961\times10^{-3}\times(50)^2 - 2.8487\times10^{-1}\times(50) + 9.6778 = 1.2784$$

$$f(x_3) = f(50+25) = f(75)$$
$$f(75) = -9.1688\times10^{-6}\times(75)^3 + 2.7961\times10^{-3}\times(75)^2 - 2.8487\times10^{-1}\times(75) + 9.6778 = 0.17253$$

$$f(x_4) = f(x_n) = f(100)$$

$$f(100) = -9.1688\times10^{-6}\times(100)^3 + 2.7961\times10^{-3}\times(100)^2 - 2.8487\times10^{-1}\times(100) + 9.6778$$
$$= 0.017000$$

$$I \approx \frac{b-a}{3n}\left[f(x_0) + 4\sum_{\substack{i=1\\ i=odd}}^{n-1} f(x_i) + 2\sum_{\substack{i=2\\ i=even}}^{n-2} f(x_i) + f(x_n)\right]$$
$$\approx \frac{100-0}{3(4)}\left[f(0) + 4\sum_{\substack{i=1\\ i=odd}}^{3} f(x_i) + 2\sum_{\substack{i=2\\ i=even}}^{2} f(x_i) + f(100)\right]$$
$$\approx \frac{100}{12}\left[f(0) + 4f(x_1) + 4f(x_3) + 2f(x_2) + f(100)\right]$$
$$\approx \frac{25}{3}\left[f(0) + 4f(25) + 4f(75) + 2f(50) + f(100)\right]$$
$$\approx \frac{25}{3}\left[0 + 4(0) + 4(0.17253) + 2(1.2784) + (-0.017000)\right]$$
$$\approx 26.917$$

b) The exact value of the above integral is found using Maple for calculating the true error and relative true error.

$$I = \int_0^{100} f(x)dx$$
$$= 60.793$$

so the true error is

$$E_t = True\ Value - Approximate\ Value$$
$$= 60.793 - (26.917)$$
$$= 33.873$$

c) The absolute relative true error, $|\in_t|$, would then be

$$|\in_t| = \left|\frac{\text{True Error}}{\text{True Value}}\right| \times 100\,\%$$
$$= \left|\frac{60.793 - (26.917)}{60.793}\right| \times 100\,\%$$
$$= 55.724\,\%$$

Table 1 Values of Simpson's 1/3 Rule for Example 2 with multiple segments.

n	Approximate Value	E_t	$\left\|\in_t\right\|$ %
2	84.947	−24.154	39.732
4	26.917	33.876	55.724
6	66.606	−5.8138	9.5633
8	62.318	−1.5252	2.5088
10	85.820	−25.023	41.169