Chapter 07.03
Simpson's 1/3 Rule for Integration-More Examples
Electrical Engineering

Example 1

All electrical components, especially off-the-shelf components do not match their nominal value. Variations in materials and manufacturing as well as operating conditions can affect their value. Suppose a circuit is designed such that it requires a specific component value, how confident can we be that the variation in the component value will result in acceptable circuit behavior? To solve this problem a probability density function is needed to be integrated to determine the confidence interval. For an oscillator to have its frequency within 5% of the target of 1 kHz, the likelihood of this happening can then be determined by finding the total area under the normal distribution for the range in question:

$$(1-\alpha)=\int_{-2.15}^{2.9}\frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}dx$$

a) Use Simpson's 1/3 Rule to find the frequency.
b) Find the true error, E_t, for part (a).
c) Find the absolute relative true error, $|\in_t|$, for part (a).

Solution

a)
$$(1-\alpha)\approx\frac{b-a}{6}\left[f(a)+4f\left(\frac{a+b}{2}\right)+f(b)\right]$$

$$a=-2.15$$
$$b=2.9$$
$$\frac{a+b}{2}=0.37500$$
$$f(x)=\frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}$$
$$f(-2.15)=\frac{1}{\sqrt{2\pi}}e^{-\frac{(-2.15)^2}{2}}$$
$$=0.039550$$
$$f(2.9)=\frac{1}{\sqrt{2\pi}}e^{-\frac{(2.9)^2}{2}}$$
$$=0.0059525$$

$$f(0.375)=\frac{1}{\sqrt{2\pi}}e^{-\frac{(0.375)^2}{2}}$$
$$=0.37186$$
$$(1-\alpha)\approx\frac{b-a}{6}\left[f(a)+4f\left(\frac{a+b}{2}\right)+f(b)\right]$$
$$\approx\left(\frac{(2.9-(-2.15))}{6}\right)[f(-2.15)+4f(0.37500)+f(2.9)]$$
$$\approx\left(\frac{5.05}{6}\right)[0.039550+4(0.37186)+0.0059525]$$
$$\approx 1.2902$$

b) The exact value of the above integral cannot be found. For calculating the true error and relative true error, we assume the value obtained by adaptive numerical integration using Maple as the exact value.

$$(1-\alpha)=\int_{-2.15}^{2.9}\frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}dx$$
$$=0.98236$$

So the true error is

$$E_t = True\ Value - Approximate\ Value$$
$$=0.98236-1.2902$$
$$=-0.30785$$

Absolute Relative true error,

$$|\in_t|=\left|\frac{\text{True Error}}{\text{True Value}}\right|\times 100\,\%$$
$$=\left|\frac{-0.30785}{0.98236}\right|\times 100\,\%$$
$$=31.338\,\%$$

Example 2

All electrical components, especially off-the-shelf components do not match their nominal value. Variations in materials and manufacturing as well as operating conditions can affect their value. Suppose a circuit is designed such that it requires a specific component value, how confident can we be that the variation in the component value will result in acceptable circuit behavior? To solve this problem a probability density function is needed to be integrated to determine the confidence interval. For an oscillator to have its frequency within 5% of the target of 1 kHz, the likelihood of this happening can then be determined by finding the total area under the normal distribution for the range in question:

$$(1-\alpha)=\int_{-2.15}^{2.9}\frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}dx$$

a) Use four segment Simpson’s 1/3 Rule to find the frequency.
b) Find the true error, E_t, for part (a).

c) Find the absolute relative true error for part (a).

Solution

a) Using n segment Simpson's 1/3 Rule,

$$(1-\alpha) \approx \frac{b-a}{3n}\left[f(x_0)+4\sum_{\substack{i=1\\i=odd}}^{n-1} f(x_i)+2\sum_{\substack{i=2\\i=even}}^{n-2} f(x_i)+f(x_n)\right]$$

$$n = 4$$
$$a = -2.15$$
$$b = 2.9$$
$$h = \frac{b-a}{n}$$
$$= \frac{2.9-(-2.15)}{4}$$
$$= 1.2625$$
$$f(x) = \frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}$$

So

$$f(x_0) = f(-2.15)$$
$$f(-2.15) = \frac{1}{\sqrt{2\pi}}e^{-\frac{(-2.15)^2}{2}}$$
$$= 0.03955$$

$$f(x_1) = f(-2.15+1.265)$$
$$= f(-0.8875)$$
$$= \frac{1}{\sqrt{2\pi}}e^{-\frac{(-0.8875)^2}{2}}$$
$$= 0.26907$$

$$f(x_2) = f(-0.8875+1.2625)$$
$$= f(0.375)$$
$$= \frac{1}{\sqrt{2\pi}}e^{-\frac{(0.375)^2}{2}}$$
$$= 0.37186$$

$$f(x_3) = f(0.375+1.2625)$$
$$= f(1.6375)$$
$$= \frac{1}{\sqrt{2\pi}}e^{-\frac{(1.6375)^2}{2}}$$

$$= 0.10439$$

$$\begin{aligned} f(x_4) &= f(x_n) \\ &= f(2.9) \\ &= \frac{1}{\sqrt{2\pi}} e^{-\frac{(2.9)^2}{2}} \\ &= 0.0059525 \end{aligned}$$

$$\begin{aligned} (1-\alpha) &\approx \frac{b-a}{3n}\left[f(x_0) + 4\sum_{\substack{i=1 \\ i=odd}}^{n-1} f(x_i) + 2\sum_{\substack{i=2 \\ i=even}}^{n-2} f(x_i) + f(x_n) \right] \\ &\approx \frac{2.9-(-2.15)}{3(4)}\left[f(-2.15) + 4\sum_{\substack{i=1 \\ i=odd}}^{3} f(x_i) + 2\sum_{\substack{i=2 \\ i=even}}^{2} f(x_i) + f(2.9) \right] \\ &\approx \frac{5.05}{12}\left[f(-2.15) + 4f(x_1) + 4f(x_3) + 2f(x_2) + f(2.9) \right] \\ &\approx \frac{5.05}{12}\left[f(-2.15) + 4f(-0.8875) + 4f(1.6375) + 2f(0.375) + f(2.9) \right] \\ &\approx \frac{5.05}{12}\left[0.03955 + 4(0.26907) + 4(0.10439) + 2((0.37186)) + 0.0059525 \right] \\ &\approx 0.96079 \end{aligned}$$

b) The exact value of the above integral cannot be found. For calculating the true error and relative true error, we assume the value obtained by adaptive numerical integration using Maple as the exact value.

$$\begin{aligned} (1-\alpha) &= \int_{-2.15}^{2.9} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}} dx \\ &= 0.98236 \end{aligned}$$

So the true error is

$$\begin{aligned} E_t &= True\ Value - Approximate\ Value \\ &= 0.98236 - 0.96079 \\ &= 0.021568 \end{aligned}$$

c) The absolute relative true error, $|\in_t|$, would then be

$$\begin{aligned} |\in_t| &= \left| \frac{\text{True Error}}{\text{True Value}} \right| \times 100\,\% \\ &= \left| \frac{0.021568}{0.98236} \right| \times 100\,\% \\ &= 2.1955\,\% \end{aligned}$$

Table 1 Values of Simpson's 1/3 Rule for Example 2 with multiple segments.

n	Approximate Value	E_t	$\lvert \in_t \rvert \%$
2	1.2902	−0.30785	31.338
4	0.96079	0.021568	2.1955
6	0.98168	0.00068166	0.069391
8	0.98212	0.00023561	0.023984
10	0.98226	0.000092440	0.0094101