Chapter 07.03
Simpson's 1/3 Rule for Integration-More Examples
Mechanical Engineering

Example 1

A trunnion of diameter $12.363''$ has to be cooled from a room temperature of $80°\text{F}$ before it is shrink fit into a steel hub (Figure 1).

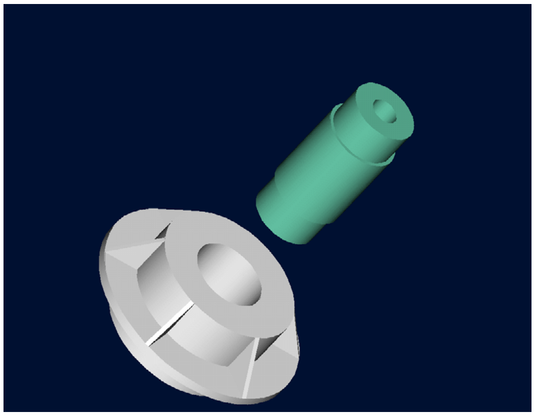

Figure 1 Trunnion to be slid through the hub after contracting.

The equation that gives the diametric contraction, in inches of the trunnion in dry-ice/alcohol (boiling temperature is $-108°\text{F}$) is given by:

$$\Delta D = 12.363\int_{80}^{-108}\left(-1.2278\times10^{-11}T^2 + 6.1946\times10^{-9}T + 6.015\times10^{-6}\right)dT$$

a) Use Use Simpson's 1/3 Rule to find the approximate value of ΔD.
b) Find the true error, E_t, for part (a).
c) Find the absolute relative true error for part (a).

Solution

a) $$\Delta D \approx \frac{b-a}{6}\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right]$$

$$a = 80$$
$$b = -108$$
$$\frac{a+b}{2} = -14$$
$$f(T) = 12.363\left(-1.2278\times10^{-11}T^2 + 6.1946\times10^{-9}T + 6.015\times10^{-6}\right)$$

$$f(80) = 12.363\left(-1.2278\times10^{-11}(80)^2 + 6.1946\times10^{-9}(80) + 6.015\times10^{-6}\right)$$
$$= 7.9519\times10^{-5}$$
$$f(-108) = 12.363\left(-1.2278\times10^{-11}(-108)^2 + 6.1946\times10^{-9}(-108) + 6.015\times10^{-6}\right)$$
$$= 6.4322\times10^{-5}$$
$$f(-14) = 12.363\left(-1.2278\times10^{-11}(-14)^2 + 6.1946\times10^{-9}(-14) + 6.015\times10^{-6}\right)$$
$$= 7.3262\times10^{-5}$$
$$\Delta D \approx \left(\frac{b-a}{6}\right)\left[f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right]$$
$$\approx \left(\frac{-108-80}{6}\right)\left[f(80) + 4f(-14) + f(-108)\right]$$
$$\approx \left(\frac{-188}{6}\right)\left[7.9519\times10^{-5} + 4\left(7.3262\times10^{-5}\right) + 6.4322\times10^{-5}\right]$$
$$\approx -0.013689 \text{ in}$$

b) The exact value of the above integral is

$$\Delta D = 12.363\int_{80}^{-108}\left(-1.2278\times10^{-11}T^2 + 6.1946\times10^{-9}T + 6.015\times10^{-6}\right)dT$$
$$= -0.013689 \text{ in}$$

so the true error is

$$E_t = \text{True Value} - \text{Approximate Value}$$
$$= -0.013689 - (-0.013689)$$
$$= 0.0000$$

c) The absolute relative true error, $|\in_t|$, would then be

$$|\in_t| = \left|\frac{\text{True Error}}{\text{True Value}}\right| \times 100\,\%$$
$$= \left|\frac{0.0000}{-0.013689}\right| \times 100\,\%$$
$$= 0.0000\,\%$$

Example 2

A trunnion of diameter 12.363″ has to be cooled from a room temperature of 80°F before it is shrink fit into a steel hub (Figure 1). The equation that gives the diametric contraction, in inches of the trunnion in dry-ice/alcohol (boiling temperature is -108°F) is given by:

$$\Delta D = 12.363\int_{80}^{-108}\left(-1.2278\times10^{-11}T^2 + 6.1946\times10^{-9}T + 6.015\times10^{-6}\right)dT$$

a) Use four segment Simpson's 1/3 Rule to find the contraction.
b) Find the true error, E_t, for part (a).

c) Find the absolute relative true error for part (a).

Solution

a) $\Delta D = \frac{b-a}{3n}\left[f(T_0) + 4\sum_{\substack{i=1 \\ i=odd}}^{n-1} f(T_i) + 2\sum_{\substack{i=2 \\ i=even}}^{n-2} f(T_i) + f(T_n)\right]$

$$n = 4$$
$$a = 80$$
$$b = -108$$
$$h = \frac{b-a}{n}$$
$$= \frac{-108-80}{4}$$
$$= -47$$
$$f(T) = 12.363\left(-1.2278\times10^{-11}T^2 + 6.1946\times10^{-9}T + 6.015\times10^{-6}\right)$$
$$\Delta D \approx \frac{b-a}{3n}\left[f(T_0) + 4\sum_{\substack{i=1 \\ i=odd}}^{n-1} f(T_i) + 2\sum_{\substack{i=2 \\ i=even}}^{n-2} f(T_i) + f(T_n)\right]$$
$$\approx \frac{-108-80}{3(4)}\left[f(80) + 4\sum_{\substack{i=1 \\ i=odd}}^{3} f(T_i) + 2\sum_{\substack{i=2 \\ i=even}}^{2} f(T_i) + f(-108)\right]$$
$$\approx \frac{-188}{12}\left[f(80) + 4f(T_1) + 4f(T_3) + 2f(T_2) + f(-108)\right]$$
$$\approx \frac{-188}{12}\left[f(80) + 4f(33) + 4f(-61) + 2f(-14) + f(-108)\right]$$
$$\approx -15.667\begin{bmatrix}7.9519\times10^{-5} + 4\left(7.6725\times10^{-5}\right) + 4\left(7.0257\times10^{-5}\right) \\ + 2\left(7.3321\times10^{-5}\right) + 6.4322\times10^{-5}\end{bmatrix}$$
$$\approx -0.013689 \text{ in}$$

Since

$$f(T_0) = f(80)$$
$$= 12.363\left(-1.2278\times10^{-11}(80)^2 + 6.1946\times10^{-9}(80) + 6.015\times10^{-6}\right)$$
$$= 7.9519\times10^{-5}$$

$$f(T_1) = f(80-47)$$
$$= f(33)$$
$$= 12.363\left(-1.2278\times10^{-11}(33)^2 + 6.1946\times10^{-9}(33) + 6.015\times10^{-6}\right)$$
$$= 7.6725\times10^{-5}$$

$$f(T_2) = f(33-47)$$

$$= f(-14)$$
$$= 12.363\left(-1.2278\times 10^{-11}(-14)^2 + 6.1946\times 10^{-9}(-14) + 6.015\times 10^{-6}\right)$$
$$= 7.3262\times 10^{-5}$$

$$f(T_3) = f(-14-47)$$
$$= f(-61)$$
$$= 12.363(-1.2278\times 10^{-11}(-61)^2 + 6.1946\times 10^{-9}(-61) + 6.015\times 10^{-6})$$
$$= 7.0257\times 10^{-5}$$

$$f(T_4) = f(T_n)$$
$$= f(-108)$$
$$= 12.363\left(-1.2278\times 10^{-11}(-108)^2 + 6.1946\times 10^{-9}(-108) + 6.015\times 10^{-6}\right)$$
$$= 6.4322\times 10^{-5}$$

b) The exact value of the above integral is

$$\Delta D = 12.363\int_{80}^{-108}\left(-1.2278\times 10^{-11}T^2 + 6.1946\times 10^{-9}T + 6.015\times 10^{-6}\right)dT$$

$$= -0.013689 \text{ in}$$

so the true error is

$$E_t = \text{True Value} - \text{Approximate Value}$$
$$= -0.013689 - (-0.013689)$$
$$= 0.0000$$

c) The absolute relative true error, $|\in_t|$, would then be

$$|\in_t| = \left|\frac{\text{True Error}}{\text{True Value}}\right|\times 100\,\%$$
$$= \left|\frac{0.0000}{-0.013689}\right|\times 100\,\%$$
$$= 0.0000\,\%$$

Table 1 Values of Simpson's 1/3 Rule for Example 2 with multiple segments.

n	Approximate Value	E_t	$\lvert\in_t\rvert$ %
2	−0.013689	0.0000	0.0000
4	−0.013689	0.0000	0.0000
6	−0.013689	0.0000	0.0000
8	−0.013689	0.0000	0.0000
10	−0.013689	0.0000	0.0000